Extensionality of Spatial Observations in Distributed Systems

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Abstract

We discuss the tensions between intensionality and extensionality of spatial observations in distributed systems, showing that there are natural models where extensional observational equivalences may be characterized by spatial logics, including the composition and void operators. Our results support the claim that spatial observations do not need to be always considered intensional, even if expressive enough to talk about the structure of systems. For simplicity, our technical development is based on a minimalist process calculus, that already captures the main features of distributed systems, namely local synchronous communication, local computation, asynchronous remote communication, and partial failures.

Introduction

Logical characterizations of concurrent behaviors have been introduced for a long time now. A fundamental result in the field, due to Hennessy and Milner [12], is the characterization of behavioral equivalence in process algebras as indistinguishability with respect to a modal logic. Such results are important not only theoretically, but also because of their influence in the design of practical specification languages for software systems. Hennessy-Milner logic (HML) adds to propositional operators the action modality $\langle\lambda\rangle A$, allowing the logic to observe a grain of behavior: a process satisfies $\langle\lambda\rangle A$ if it satisfies A after performing action λ. HML characterizes behavioral equivalence in the sense that two processes are strongly bisimilar if and only if they satisfy exactly the same formulas.

More recently, spatial logics for concurrency [6, 8, 4] have been proposed with the aim of specifying distributed behavior and other essential aspects of distributed computing systems. In general terms, these developments reflect a shift of focus in concurrency research, that has been building up from the last decade on, from the study of centralized concurrent systems to the study of general distributed systems. While centralized processes may be accurately modeled as pure objects of behavior, in distributed systems many interesting phenomena besides pure interaction, such as location dependent behavior, resource usage, and mobility, must be considered.

Present in all spatial logics for concurrency are the composition operator $A \mid B$ and the void operator $\mathbf{0}$ [4]. Intuitively, a system satisfies $A \mid B$ if it can be decomposed in two disjoint subsystems such that one satisfies A and the other satisfies B, while a system satisfies $\mathbf{0}$ if it is the empty system. The guarantee (logical adjunct of the composition operator) $A \triangleright B$, introduced in [8], allows the logic to talk about contextual properties. Namely, a process satisfies $A \triangleright B$ if whenever composed with a system that satisfies A, yields a (possibly larger) system that satisfies B. Decomposition and composition of systems as mentioned here is generally interpreted up to structural congruence, and thus structural congruence seems to play a key role in the semantics of spatial logics.

Observation of features such as spatial separation are frequently considered intensional because they usually induce fine distinctions among processes that are not substantiated by purely behavioral (extensional) observations. According to Sangiorgi [18], “A logic is intensional if it can separate terms on the basis of their internal structure, even though their behaviors are the same”. Moreover, in many situations, it turns out that the logical equivalence induced by a spatial logic on processes, is not only strictly finer than behavioral congruence, but coincides with structural congruence [18, 5, 10, 19].

These results contributed to widespread the impression that spatial observations, as those induced by spatial connectives, are intrinsically intensional, imposed extraneously so to increase the power of the observer. For example, Hirschkoff has shown [13] that if the so-called intensional connectives composition and void are removed from a spatial logic for the pi-calculus, while retaining the guarantee, one obtains a logic whose separation power precisely coincides with strong bisimulation and may then be considered extensional. The ability of the spatial connectives to capture structural congruence is also attributed to their ability to count, separate, and express arithmetical constraints, *e.g.*, about the number of subsystems of a given system. The observational power of spatial logics may then sometimes appear a bit arbitrary, in the sense that structural congruence does not have a canonical status among behavioral process equivalences, and is frequently seen just as a technical convenience, with a syntactic flavor, to ease the presentation of a calculus operational semantics.

On the other hand, it has been argued [4, 2, 3] that the intensional character of logical characterizations of spatiality in distributed computation may be, at least in part, incidental, and does not necessarily reflect the fundamental motivation for introducing spatial logics for concurrency. Ideally, we would like spatial observations, as captured by spatial logics, to reflect natural distinctions and similarities between distributed systems, in a context where spatial location is a relevant observable, in parity with more standard behavioral observables. We expect spatial observations of the sort, captured by spatial logic operators such as composition, to be taken modulo an intended notion of equality of the observable space-time structure, independently on whether such equality relation is technically defined using a notion of structural congruence. If certain spatial-behavioral observations precisely capture the observable structure of a model in our sense, they would have to be considered extensional, even if able to detect aspects of spatial structure.

In this paper, we pursue the informal discussion started above in technical terms. Namely, we make precise the claim that spatial observations, including structural ones, may be understood as purely extensional in fairly natural models of distributed systems. To discuss the several issues of interest in a simplified setting, we consider a minimal distributed process calculus, obtained by extending the smallest concurrent fragment of CCS with flat anonymous locations. Our model can be seen as a general abstraction of the essence of distributed systems, already featuring all the key ingredients present in distributed process calculi, although in a possibly less refined way. Processes may synchronously communicate locally to a site through standard CCS-like synchronization, and asynchronously communicate at a distance, by means of a migration primitive. We also allow systems to nondeterministically exhibit partial failures, as in [1, 11]. Notice that it is not our aim here to propose yet another distributed process calculus, but rather to set up a convenient setting to compare distributed system observational equivalences and their spatial logical characterizations.

Our technical contributions may be summarized as follows. After introducing the process calculus and its reduction semantics, we define observational equivalence by adopting the canonical notion of reduction barbed congruence. Barbed congruence [16] and reduction barbed congruence [15] are currently accepted as the standard approach to define reference behavioral equivalences for general process calculi. After showing some basic properties of reduction barbed congruence in our setting, we define strong bisimulation, an alternative coinductive characterization of observational equivalence, which is shown equivalent to reduction barbed congruence. The interesting aspect of our definition of strong bisimulation is that it contains "intensional" clauses (in the sense of [18]), namely a clause expressing separation, and a clause for observing the empty system. We then use the characterization of reduction barbed congruence in terms of strong bisimulation to identify a spatial logic characterization of both reduction barbed congruence and strong bisimulation: our logic is an extension of HML with the composition and void operators of spatial logic. The same line of development is also carried out for the weak case. In this latter setting, we prove minimality of the logic, thus showing the essential role of all of the logic operators, in particular of the spatial operators, in the intended expressive and separation power. We can verify that in both the strong and weak cases the process equivalences induced by the logics are coarser than structural congruence, and that the presence of the composition and void operators, semantically interpreted in the standard way, do not carry any lack of extensionality (with extensionality interpreted with reference to a standard observational equivalence), even if the logics can express separation and counting constraints on the structure of systems.

1 A Simple Model of Distributed Systems

In this section we present the syntax and operational semantics of our distributed process calculus. Assume given an infinite set Λ of *names*, ranged over by a, b, c.

Definition 1.1 (Actions, Processes and Networks) *The sets $\mathcal{A}$ of* actions*, $\mathcal{P}$ of* processes*, and $\mathcal{N}$ of* networks *are given by:*

$$\alpha, \beta ::= \bar{a} \;\Big|\; a \;\Big|\; \tau \qquad\qquad P, Q, R ::= \mathbf{nil} \;\Big|\; P \mid Q \;\Big|\; \alpha.P \;\Big|\; \mathbf{go}.P$$
$$N, M, O ::= \mathbf{0} \;\Big|\; N \mid M \;\Big|\; [P]$$

For process actions we consider the output $\bar{a}$, the input a and the internal computation τ. For processes, we consider the smallest fragment of CCS featuring some form of concurrency, thus we have inaction **nil**, parallel composition $P \mid Q$, and action prefixing $\alpha.P$. On top of this, we introduce a notion of distribution by locating processes P inside sites of the form $[P]$, and by adding the migration capability $\mathbf{go}.P$ to processes. For the sake of simplicity, sites are not natively named, so that the $\mathbf{go}.P$ primitive allows a process to non-deterministically migrate to some other site. Thus, a distributed system is represented by a network consisting of a collection of sites spread in space by means of spatial composition $N \mid M$. The empty network is represented by $\mathbf{0}$. Interaction through CCS-like channel synchronization is only possible locally to a site. Remote communication between sites is then captured through the use of the migration primitive $\mathbf{go}.P$. We now present the operational semantics of our calculus, which is captured by the relations of structural congruence and reduction.

Definition 1.2 (Structural congruence) Structural congruence*, noted $\equiv$, is the least congruence on processes and networks such that*

$$\begin{array}{lll} P \mid \mathbf{nil} \equiv P & P \mid Q \equiv Q \mid P & P \mid (Q \mid R) \equiv (P \mid Q) \mid R \\ N \mid \mathbf{0} \equiv N & N \mid M \equiv M \mid N & N \mid (M \mid O) \equiv (N \mid M) \mid O \\ & & P \equiv Q \implies [P] \equiv [Q] \end{array}$$

Definition 1.3 (Reduction) Reduction*, noted $N \to M$, is the relation between processes inductively defined as follows*

$$[\bar{a}.P \mid a.Q \mid R] \to [P \mid Q \mid R] \text{ (Red Comm)} \qquad [\tau.P \mid Q] \to [P \mid Q] \text{ (Red Tau)}$$
$$[\mathbf{go}.P \mid Q] \mid [R] \to [Q] \mid [P \mid R] \text{ (Red Go)} \qquad [P] \mid N \to \mathbf{0} \text{ (Red Fail)}$$
$$\frac{N \to N'}{N \mid M \to N' \mid M} \text{ (Red Cong)} \qquad \frac{N \equiv N' \to M' \equiv M}{N \to M} \text{ (Red Struct)}$$

The rule (Red Comm) specifies interaction between two processes through co-action synchronization locally inside a site, while rule (Red Tau) specifies internal action of a process. Rule (Red Go) specifies that a process prefixed by **go** may migrate to another site. Rule (Red Fail) expresses that any non-empty network may fail, thus modeling fail-stop failure of an arbitrary subsystem. We believe that this operational semantics abstracts in a sensible way the essential features of any realistic distributed computing system: local synchronous communication and computation, asynchronous remote communication, and partial failures.

Our aim now is to define a natural notion of observational equivalence on networks. To that end, we adopt the canonical notion of barbed equivalence, according to which two systems are observationally equivalent if no context can distinguish between them by detecting barbs. Although we could have postulated a more refined notion of barb observation, we prefer to strictly follow the standard definition [16], even if it assumes in a sense the existence of a global observer, which is possibly debatable in the context of distributed systems. In our case, we restrict to one-hole spatial contexts, as has been usually adopted in the case of many distributed process calculi, e.g., [1, 11].

Definition 1.4 (Contexts) Contexts *are defined as follows:* $C[\bullet] ::= N \mid \bullet$.

Definition 1.5 (Barb) *A network N exhibits* barb a*, noted $N{\downarrow_a}$, if there are P, Q, M such that $N \equiv [a.P \mid Q] \mid M$.*

Presence of a barb reflects the fact that any external observer can get to know that an input is ready via some channel name, at some accessible site. Given barbs and contexts, the following gives our reference observational equivalence relation.

Definition 1.6 (Strong reduction barbed congruence) Strong reduction barbed congruence*, noted $\simeq$, is the largest symmetric relation R such that for all $(N, M) \in R$ we have*

For all barbs a, if $N{\downarrow_a}$ then $M{\downarrow_a}$	(Barb closed)
If $N \to N'$ then there is M' s.t. $M \to M'$ and $(N', M') \in R$	(Reduction closed)
For all contexts $C[\bullet]$, $(C[N], C[M]) \in R$	(Context closed)

We now establish some standard properties of strong reduction barbed congruence. Notice that we just consider in this paper congruences under spatial (static) contexts (Definition 1.4). As explained above, this does not carry a lack of generality, given the main motivations of our development.

Proposition 1.7 *$\simeq$ is a congruence. Moreover, $\equiv \subset \simeq$.*

Proof. The proof follows standard lines. To prove that $\equiv$ is strictly included in $\simeq$ we may show that $[a.\mathbf{nil} \mid a.\mathbf{nil}] \simeq [a.a.\mathbf{nil}]$ but $[a.\mathbf{nil} \mid a.\mathbf{nil}] \not\equiv [a.a.\mathbf{nil}]$. ■

We use $fn(P)$ to denote the set of free names of a process P, defined as usual, and the same for a network N. To bring the presentation more readable, we introduce the following abbreviations for certain collections of processes and sites.

$$\prod_{i \in I} P_i \triangleq P_1 \mid \ldots \mid P_l \text{ where } I = \{1, \ldots, l\}$$

$$\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right] \triangleq \left[\prod_{i\in I_1} P_i^1\right] \mid \ldots \mid \left[\prod_{i\in I_k} P_i^k\right] \text{ where } J = \{1,\ldots,k\}$$

N.B.: If $I_j = \emptyset$ then $\left[\prod_{i\in I_j} P_i^j\right] \equiv [\mathbf{nil}]$ and if $J = \emptyset$ then $\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right] \equiv \mathbf{0}$.

It follows from Proposition 1.7 that strong reduction barbed congruent networks compose to yield strong reduction barbed congruent networks. In particular, we have:

Lemma 1.8 *Let P^i and Q^i $(i \in J)$ be collections of processes. If for all $i \in J$ we have $\left[P^i\right] \simeq \left[Q^i\right]$, then also $\prod_{j\in J}\left[P^j\right] \simeq \prod_{j\in J}\left[Q^j\right]$.*

Although observational equivalence (Definition 1.6) is defined in the standard way, with reference to the global observation of barbs in networks, observations already leak some relevant information about the distributed structure of systems, as an effect, in our case, of the combination of migration capabilities with the failure model. Our next lemma states that strong reduction barbed congruent networks always result from an underlying one-one and onto correspondence of strong reduction barbed congruent sites. In particular, we conclude that strong reduction barbed congruent networks always have the same number of sites.

Lemma 1.9 *Let M, N be networks such that $N \triangleq \prod_{j\in J}\left[P^j\right]$, where P^j $(j \in J)$ is a collection of processes, and $N \simeq M$. Then there is a collection of processes Q^j $(j \in J)$ such that $M \equiv \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$ we have $\left[P^j\right] \simeq \left[Q^j\right]$.*

Proof. (Sketch, full proof in appendix) We consider a context of the form

$$C\left[\bullet\right] \triangleq \left[t.\mathbf{nil} \mid \prod_{i\in J}\mathbf{go}.(f_i.\mathbf{nil} \mid \prod_{k\in J\setminus\{i\}}\bar{f}_k.fail.\mathbf{nil})\right] \mid \bullet$$

using names such that $(\{t, fail\} \cup \{f_i \mid i \in J\}) \cap fn(N \mid M) = \emptyset$ and $(\{t, fail\} \cup \{f_i \mid i \in J\})$ are pairwise distinct. Context $C\left[\bullet\right]$ holds processes that may migrate and *mark* every site of N with an input on the unique name f_i. We make sure that every input is marking a different site by equipping the migrating process with outputs on every other f_i, and a continuation with an input on $fail$ to flag a possible synchronization. Since M behaves the same as N under $C\left[\bullet\right]$, we are sure there are at least $\#J$ sites in M, and using the symmetric reasoning we obtain that M must have $\#J$ sites. Given this, we exploit failures in N that leave only a single site active, being that this behavior must be mimicked by failures in M that also leave just one site up. These singled out sites are reduction barbed congruent, hence hold the same f_i input, thus ensuring an unique correspondence. We then consider another context that may garbage collect the marker and all the other elements we introduced, which then allows us to conclude the sites were originally strong reduction barbed congruent. ■

2 Strong Bisimilarity

Since strong reduction barbed congruence relies on universal quantification over all contexts, we now propose a more manageable characterization of observational equivalence. More concretely, we introduce a labeled transition system with the aim of capturing the contextual behavior of the networks, by means of observing process commitments, in turn expressed by transition labels. Building on such labeled transition system, a coinductive definition of bisimilarity is then presented.

Definition 2.1 (Transition labels) *The set $\mathcal{L}$ of* transition labels*, ranged over by λ, is given by $\mathcal{L} \triangleq \{\alpha \mid \alpha \in \mathcal{A}\} \cup \{[a] \mid a \in \Lambda\}$.*

Transition labels reflect internal computation (τ), and output and input communication ($\bar{a}$ and a). Notice that although communication is always local to a site, we must take into account that processes may migrate to another site and then communicate locally. Thus we also consider $[a]$ transitions, that will be used to observe migration of processes to the external environment. This turns out to be essential for covering the case of networks with a single site, since the enlargement of the system with a new site gives processes intending to migrate a possible destination. Given these ingredients, we define our labeled transition system as follows.

Definition 2.2 (Commitment) Commitment, *noted* $N \xrightarrow{\lambda} M$, *is the relation on processes and labels inductively defined as follows*

$$[\bar{a}.P \mid a.Q \mid R] \xrightarrow{\tau} [P \mid Q \mid R] \text{ (Comm)} \qquad [\tau.P \mid Q] \xrightarrow{\tau} [P \mid Q] \text{ (Tau)}$$
$$[\bar{a}.P \mid Q] \xrightarrow{\bar{a}} [P \mid Q] \text{ (Out)} \qquad [a.P \mid Q] \xrightarrow{a} [P \mid Q] \text{ (In)}$$
$$[\mathbf{go}.P \mid Q] \mid [R] \xrightarrow{\tau} [Q] \mid [P \mid R] \text{ (Go)}$$
$$[P] \mid N \xrightarrow{\tau} \mathbf{0} \text{ (Fail)} \qquad N \xrightarrow{[a]} N \mid [a.\mathbf{nil}] \text{ (Grow)}$$
$$\frac{N \xrightarrow{\lambda} N'}{N \mid M \xrightarrow{\lambda} N' \mid M} \text{ (Cong)} \qquad \frac{N \equiv N' \xrightarrow{\lambda} M' \equiv M}{N \xrightarrow{\lambda} M} \text{ (Struct)}$$

As a sanity check, we ensure that τ commitments match reductions and inversely.

Lemma 2.3 *We have* $N \xrightarrow{\tau} M$ *if and only if* $N \to M$.

Notice that although *e.g.*, the systems $[\mathbf{nil}] \mid [\mathbf{nil}]$ and $[\tau.\mathbf{nil}]$ have exactly the same commitment graph, they are not observationally equivalent in the light of Lemma 1.9. Thus, in order to properly capture strong reduction barbed congruence, we include in the definition of strong bisimulation two spatial clauses (referred to as "intensional clauses" in [18]). We then have

Definition 2.4 (Strong Bisimulation) *A binary relation* $B \subseteq \mathcal{N} \times \mathcal{N}$ *is a* strong bisimulation *if and only if it is symmetric and whenever* $(N, M) \in B$ *then*

$$\begin{array}{rcl} N \equiv N' \mid N'' & \Rightarrow & \exists M', M'' \,.\, M \equiv M' \mid M'' \wedge (N', M') \in B \wedge (N'', M'') \in B \\ N \equiv \mathbf{0} & \Rightarrow & M \equiv \mathbf{0} \\ N \xrightarrow{\lambda} N' & \Rightarrow & \exists M' \,.\, M \xrightarrow{\lambda} M' \wedge (N', M') \in B \end{array}$$

Notice that the second clause in Definition 2.4 is subsumed by the third one since (due to the (Fail) transition) only void systems have no possible internal actions, however we prefer to include it in the definition for the sake of uniformity with the corresponding weak version, and thus avoid some extra incidentality. Notice also that the first clause properly distinguishes $[\tau.\mathbf{nil}]$ and $[\mathbf{nil}] \mid [\mathbf{nil}]$, because there is no way to split $[\tau.\mathbf{nil}]$ (up to $\equiv$) in two components with some transition each. We collect some basic results about strong bisimulation.

Proposition 2.5 *Strong bisimulations are equivalence relations.*

Proposition 2.6 *Let* S *be a set of strong bisimulations. Then* $\bigcup S$ *is a strong bisimulation.*

We thus define

Definition 2.7 (Strong bisimilarity) Strong bisimilarity, *noted* $\sim$, *is the greatest strong bisimulation.*

Proposition 2.8 *We have* $\equiv \subset \sim$.

Proof. That $\equiv$ is contained in $\sim$ follows by a standard coinductive argument. To see that the inclusion is strict, notice that although $[a.\mathbf{nil} \mid a.\mathbf{nil}] \sim [a.a.\mathbf{nil}]$ it is not the case that $[a.\mathbf{nil} \mid a.\mathbf{nil}] \equiv [a.a.\mathbf{nil}]$. ∎

2.1 Full Abstraction

This section is devoted to proving that strong bisimilarity, as defined in Definition 2.7, characterizes strong reduction barbed congruence in a fully abstract way. The proof builds on a series of intermediate technical results.

Lemma 2.9 *Let M be a network and $P^j (j \in J)$ a collection of processes where $\prod_{j\in J}\left[P^j\right] \sim M$. Then there is a collection of processes $Q^j (j \in J)$ such that $M \equiv \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$, $\left[P^j\right] \sim \left[Q^j\right]$.*

Proof. By induction on the size of J, using the first two clauses in the definition of the strong bisimulation. ■

The proof of the main result of this section (Theorem 2.14) is not technically involved, but critically depends on next Lemma 2.10, that expresses a key compositionality principle of our calculus. Notice that the basic building block of systems referred to in the statement of Lemma 2.10 is the process: since we have to take migration into account, it is essential to assure compositionality at the process level.

Lemma 2.10 *Let J be a finite set and I_j, for all $j \in J$, be a finite set. Let P_i^j and Q_i^j be processes such that for all $j \in J$ and $i \in I_j$ we have $\left[P_i^j\right] \sim \left[Q_i^j\right]$. Then*

$$\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right] \sim \prod_{j\in J}\left[\prod_{i\in I_j} Q_i^j\right]$$

Proof. (Sketch, full proof in appendix) By coinduction on the definition of strong bisimulation. We sketch the proof for the interesting case of migration.

When considering that a migration may take place, originating in some P_l^k, we can induce a grow transition $[a]$ using fresh name a, being fresh in the sense that it does not occur in neither one of the P_i^js and Q_i^js, after which we know that the migration may target the newly created site. To mimic this behavior there must be two steps made by Q_l^k, which we know to be bisimilar to P_l^k, being the first one the $[a]$ transition that creates the site, and the second a τ action after which the sites that contain a must be bisimilar. This is so, because we can decompose and induce a transition on a, and also obtain that the site's contents is whatever process migrated from P_i^j, whose behavior must match the remaining content of the corresponding site. Notice that we can not be sure that a migration on one side is always matched by a migration on the other, because the migrating process can *e.g.*, be inaction. In that case, a migration may be matched by an internal computation step. To finish up, we can chose the destination of the migration to be any $m \in J\backslash\{k\}$ and be sure to obtain a collection of sites that respect the statement of the Lemma. ■

Given the previous results we can now prove strong bisimilarity is a congruence.

Lemma 2.11 *Strong bisimilarity is a congruence.*

Proof. We use Lemma 2.9 to break two strongly bisimilar networks N and M down to bisimilar sites. We then consider Lemma 2.10 that gives us that the composition of the bisimilar sites of N and M with the structurally congruent sites, hence bisimilar sites, of the context results in bisimilar networks, hence $C[N]$ and $C[M]$ are strongly bisimilar. ■

Using these results we prove the two parts of our full abstraction theorem.

Lemma 2.12 *We have $\sim \subseteq \simeq$.*

Proof. By coinduction on the definition of strong reduction barbed congruence. To prove that $\sim$ is barb closed we exploit the input transitions, to prove reduction closure we exploit τ transitions which coincide with reductions (Lemma 2.3), and to prove context closure we consider Lemma 2.11. ■

Lemma 2.13 *We have* $\simeq \subseteq \sim$.

Proof. By coinduction on the definition of strong bisimulation. The separation clause follows from Lemma 1.9 and Lemma 1.8 that allow us to decompose strong reduction barbed congruent networks down to strong reduction barbed congruent sites and then compose them up to the separation we require and obtain strong reduction barbed congruent networks. The emptiness clause results immediately from Lemma 1.9. For the transition clause we consider the various labels: either it is a τ transition, and in that case we consider that τ transitions match reductions (Lemma 2.3), or it is either an input or output, in which case we devise a specially crafted context that serves as a witness of the existence of that action, and finally the grow transition which is immediate since we have context closure. ■

By Lemma 2.12 and Lemma 2.13,we can state our first main result

Theorem 2.14 (Full abstraction) *We have* $\sim = \simeq$.

In the next section, we build on the characterization of reduction barbed congruence in terms of strong bisimulation stated in Theorem 2.14 to define a logical characterization of behavioral equivalence.

2.2 Logical Characterization of Strong Bisimilarity

In this section, we characterize strong bisimilarity (and thus strong reduction barbed congruence) in logical terms, using a simple spatial logic.

Definition 2.15 (Spatial logic $\mathcal{L}_s$) *Formulas are defined by the following syntax:*

$$\text{(Formulas) } A, B, C ::= \mathbf{T} \;\Big|\; \neg A \;\Big|\; A \wedge B \;\Big|\; \mathbf{0} \;\Big|\; A \mid B \;\Big|\; \langle \lambda \rangle A$$

Our logic, besides the usual action modality from HML, includes the composition and void operators of spatial logics, interpreted in the standard way. For example, we may express property "network has exactly one site" by the formula $\neg\mathbf{0} \wedge \neg(\neg\mathbf{0} \mid \neg\mathbf{0})$. The semantics of the logic is given by the denotation of the formulas, i.e., a formula denotes the set of networks that satisfy it.

Definition 2.16 (Semantics of $\mathcal{L}_s$) *A formula's denotation is inductively given by*

$$[\![\mathbf{T}]\!] \triangleq \mathcal{N} \quad [\![\neg A]\!] \triangleq \mathcal{N} \backslash [\![A]\!] \quad [\![A \wedge B]\!] \triangleq [\![A]\!] \cap [\![B]\!] \quad [\![\mathbf{0}]\!] \triangleq \{N \mid N \equiv \mathbf{0}\}$$
$$[\![A \mid B]\!] \triangleq \{N \mid \exists N', N'' \,.\, N \equiv N' \mid N'' \wedge N' \in [\![A]\!] \wedge N'' \in [\![B]\!]\}$$
$$[\![\langle \lambda \rangle A]\!] \triangleq \{N \mid \exists N' \,.\, N \xrightarrow{\lambda} N' \wedge N' \in [\![A]\!]\}$$

We write $N \models A$ to mean $N \in [\![A]\!]$. We say that networks M and N are *logically equivalent* w.r.t. $\mathcal{L}_s$, written $M =_{\mathcal{L}_s} N$, if and only if they satisfy exactly the same formulas of $\mathcal{L}_s$, namely if and only if, for any formula A of $\mathcal{L}_s$, we have that $M \models A \iff N \models A$. As with other spatial logics, it is an immediate consequence of the definition that satisfaction is closed under structural congruence.

Lemma 2.17 *We have* $\equiv \subseteq =_{\mathcal{L}_s}$.

Proof. Standard, by induction on the structure of the formulas. ■

We now prove the implications that guarantee that $=_{\mathcal{L}_s}$ characterizes $\sim$.

Lemma 2.18 *We have* $\sim \subseteq =_{\mathcal{L}_s}$.

Proof. Follows from a standard induction on the structure of the formulas. ■

Lemma 2.19 *We have* $=_{\mathcal{L}_s} \subseteq \sim$.

Proof. (Sketch, full proof in appendix) By coinduction on the definition of strong bisimulation, using the witness $R \triangleq \{(N, M) \mid N =_{\mathcal{L}_s} M\}$. Proof of the emptiness clause is immediate. For both the separation and transition clauses we build on the fact that the image set of the transition for the latter and of all possible decompositions for the former is finite (up to structural congruence). We discuss here the more interesting case of decomposition. Given a decomposition N_1, N_2 of the network $N(\equiv N_1 \mid N_2)$ we proceed, aiming at a contradiction, by assuming that a logically equivalent network M has no decomposition M_1, M_2 logically equivalent to N_1, N_2 (respectively). Hence, there is a formula that distinguishes N_1, N_2 from all elements in the set of decompositions of M. This means that for all decompositions M_1, M_2 of M there is either a formula A' for which N_1 is a model and M_1 is not, or a formula A'' for which N_2 is a model and M_2 is not. We collect this (finite) set of formulas and notice that N satisfies $(\bigwedge A') \mid (\bigwedge A'')$. Hence, M must also satisfy it and thus there is a decomposition that satisfies all formulas which leads to our intended contradiction. Thus, there exists a decomposition of M into logically equivalent (hence bisimilar, by coinduction) networks. ■

By Lemma 2.18 and Lemma 2.19, we conclude

Theorem 2.20 (Logical Characterization of $\sim$) *We have* $\sim \, = \, =_{\mathcal{L}_s}$.

Corollary 2.21 (Logical Characterization of $\simeq$) *We have* $\simeq \, = \, =_{\mathcal{L}_s}$.

We have concluded that the separation power of our spatial logic coincides with behavioral equivalence, even if it includes the basic structural connectives of composition and void, allowing it to *e.g.*, express arithmetical constraints on the number of sites in a system. We may however ask whether these structural operations are essential to characterize behavioral equivalence, in other words, whether the logic is minimal in some sense. We will give a positive answer to this question in the next section, in the more interesting case of weak behavioral equivalences.

3 Weak Bisimilarity

In this section we refine our previous results by considering a coarser observational equivalence, disregarding internal action, adopting weak reduction barbed congruence as the reference observational equivalence. We denote by $\Rightarrow$ the reflexive-transitive closure of reduction ($\rightarrow$) and define:

Definition 3.1 (Weak barb) *A network N weakly exhibits a barb a, noted $N{\Downarrow_a}$, if there is N' such that $N \Rightarrow N'$ and $N'{\downarrow_a}$.*

Definition 3.2 (Weak reduction barbed congruence) *Weak reduction barbed congruence, noted $\approxeq$, is the largest symmetric relation R such that for all $(N, M) \in R$ we have*

For all barbs a, if $N{\downarrow_a}$ then $M{\Downarrow_a}$	(Barb closed)
If $N \rightarrow N'$ then there is M' s.t. $M \Rightarrow M'$ and $(N', M') \in R$	(Reduction closed)
For all contexts $C[\bullet]$, $(C[N], C[M]) \in R$	(Context closed)

We establish a basic property of weak reduction barbed congruence, and relate it to the strong reduction barbed congruence.

Proposition 3.3 $\approxeq$ *is a congruence.*

Proof. The proof follows standard lines. ∎

Lemma 3.4 *We have* $\simeq \subset \approxeq$.

Proof. Proof of inclusion follows by a standard coinductive argument. To prove $\simeq$ is strictly included in $\approxeq$ we may show that $[\mathbf{go}.\mathbf{nil}] \approxeq [\mathbf{nil}]$ but $[\mathbf{go}.\mathbf{nil}] \not\simeq [\mathbf{nil}]$. ∎

From Proposition 3.3 we obtain that reduction barbed congruence is closed under composition, which in particular for site composition gives us:

Lemma 3.5 *Let* P^i *and* Q^i $(i \in J)$ *be collections of processes. If for all* $i \in J$ *we have* $\left[P^i\right] \approxeq \left[Q^i\right]$, *then also* $\prod_{j \in J}\left[P^j\right] \approxeq \prod_{j \in J}\left[Q^j\right]$.

As for the strong case, weak reduction barbed congruence is already able to distinguish systems based on aspects of their structure, for instance, weak reduction barbed congruent networks always have the same number of sites. Also, as stated in Lemma 3.6, weak reduction barbed congruent networks weakly reduce to a one-one and onto correspondence of weakly reduction barbed congruent sites.

Lemma 3.6 *Let* M *be a network and* P^j $(j \in J)$ *a collection of processes such that* $\prod_{j \in J}\left[P^j\right] \approxeq M$. *Then there is a collection of processes* Q^j $(j \in J)$ *such that* $M \Rightarrow \prod_{j \in J}\left[Q^j\right]$ *and for all* $j \in J$ *we have* $\left[P^j\right] \approxeq \left[Q^j\right]$.

Proof. (Sketch, full proof in appendix) The general idea is similar to that in the proof of Lemma 1.9. However, since now we may only weakly observe a barb, a different trick must be used to make sure that the migration of all the mark-placing processes has already occurred. We thus exploit the failure behavior of the context at a chosen point, avoiding in this way any chance for the migratory processes to postpone their choice of target, thus ensuring an unique correspondence. ∎

3.1 Weak Bisimilarity

As for the strong case, we now propose a coinductive characterization of weak reduction barbed congruence. We start by defining

Definition 3.7 (Weak commitment) *Weak commitment, noted* $N \stackrel{\lambda}{\Longrightarrow} N'$, *is the relation on processes and labels defined as* $N \xrightarrow{\tau}{}^{*} M' \xrightarrow{\lambda} M'' \xrightarrow{\tau}{}^{*} N'$ *for* $\lambda \neq \tau$ *or* $N \xrightarrow{\tau}{}^{*} N'$ *if* $\lambda = \tau$.

Given this we define weak bisimulations adapting the labeled transition and separation clauses to the weak case.

Definition 3.8 (Weak Bisimulation) *A binary relation* $B \subseteq \mathcal{N} \times \mathcal{N}$ *is a weak bisimulation if and only if it is symmetric and whenever* $(N, M) \in B$ *then*

$$\begin{array}{rcl} N \equiv N' \mid N'' & \Rightarrow & \exists M', M'' \,.\, M \Rightarrow M' \mid M'' \wedge (N', M') \in B \wedge (N'', M'') \in B \\ N \equiv \mathbf{0} & \Rightarrow & M \equiv \mathbf{0} \\ N \xrightarrow{\lambda} N' & \Rightarrow & \exists M' \,.\, M \stackrel{\lambda}{\Longrightarrow} M' \wedge (N', M') \in B \end{array}$$

We have the usual properties one would expect of weak bisimulations.

Proposition 3.9 *Weak bisimulations are equivalence relations.*

Proposition 3.10 *Let S be a set of weak bisimulations. Then $\bigcup S$ is a weak bisimulation.*

We can now define weak bisimilarity.

Definition 3.11 (Weak bisimilarity) Weak bisimilarity, *noted $\approx$, is the greatest weak bisimulation.*

Next Lemma 3.12 clarifies the relation between strong and weak bisimilarity.

Lemma 3.12 *We have $\sim \subset \approx$.*

Proof. The proof of inclusion follows by a standard coinductive argument. To prove strict inclusion we may show that [**go**.**nil**] $\approx$ [**nil**] but [**go**.**nil**] $\not\sim$ [**nil**]. ■

3.2 Full Abstraction

In this section, we prove that weak bisimilarity characterizes weak reduction barbed congruence in a fully abstract way, proof of which builds on the following results.

Lemma 3.13 *Let M be a network and P^j $(j \in J)$ a collection of processes such that $\prod_{j\in J}\left[P^j\right] \approx M$. Then there is a collection of processes Q^j $(j \in J)$ such that $M \Rightarrow \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$, $\left[P^j\right] \approx \left[Q^j\right]$*

Proof. By induction on the size of J, using the separation and emptiness clauses. ■

Lemma 3.14 is the cornerstone for proving full abstraction (Theorem 3.18). As for the strong case we must ensure compositionality at the process level due to process mobile capability, as their migration to sites results in the inner site composition of processes.

Lemma 3.14 *Let J be a finite set and I_j, for all $j \in J$, be a finite set. Let P_i^j and Q_i^j be two collections of processes such that for all $j \in J$ and $i \in I_j$ we have $\left[P_i^j\right] \approx \left[Q_i^j\right]$. Then also*

$$\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right] \approx \prod_{j\in J}\left[\prod_{i\in I_j} Q_i^j\right]$$

Proof. By coinduction on the definition of strong bisimulation. The proof follows the lines given for Lemma 2.10, with several adaptations needed for the weak case. Interesting to notice, in the strong case a migration of the inaction process could be mimicked by an internal computation, whilst here it can be mimicked by the empty sequence of internal actions (we no longer distinguish [**go**.**nil**] from [**nil**]). ■

Given these results we can now prove that weak bisimilarity is a congruence.

Lemma 3.15 *Weak bisimilarity is a congruence.*

Proof. By Lemma 3.13 and Lemma 3.14, along the lines of Lemma 2.11. ■

Using these results we prove the implications that provide with full abstraction.

Lemma 3.16 *We have that $\approx \subseteq \approxeq$.*

Proof. Analogous to Lemma 2.12. ■

Lemma 3.17 *We have that $\approxeq \subseteq \approx$.*

Proof. Analogous to Lemma 2.13. ■

By Lemma 3.16 and Lemma 3.17 we can now state

Theorem 3.18 (Full abstraction) *We have that $\approx = \approxeq$.*

3.3 Logical Characterization of Weak Bisimilarity

As for the strong case, we may characterize weak bisimilarity (and thus weak reduction barbed congruence) using a simple spatial logic, building on full abstraction of weak bisimilarity.

Definition 3.19 (Spatial Logic $\mathcal{L}_w$) *Formulas are defined by the following syntax:*

$$\text{(Formulas) } A, B, C ::= \mathbf{T} \;\Big|\; \neg A \;\Big|\; A \wedge B \;\Big|\; \mathbf{0} \;\Big|\; A \Uparrow B \;\Big|\; \langle\!\langle \lambda \rangle\!\rangle A$$

We adapt both the composition operator and the action modality to the weak case, while we leave the void operator with it's usual interpretation (notice that $N \Rightarrow \mathbf{0}$ is a trivial condition, due to the failure behavior).

Definition 3.20 (Semantics of $\mathcal{L}_w$) *A formula's denotation is inductively given by*

$$[\![\mathbf{T}]\!] \triangleq \mathcal{N} \quad [\![\neg A]\!] \triangleq \mathcal{N} \backslash [\![A]\!] \quad [\![A \wedge B]\!] \triangleq [\![A]\!] \cap [\![B]\!] \quad [\![\mathbf{0}]\!] \triangleq \{N \mid N \equiv \mathbf{0}\}$$
$$[\![A \Uparrow B]\!] \triangleq \{N \mid \exists N', N'' \,.\, N \Rightarrow N' \mid N'' \wedge N' \in [\![A]\!] \wedge N'' \in [\![B]\!]\}$$
$$[\![\langle\!\langle \lambda \rangle\!\rangle A]\!] \triangleq \{N \mid \exists N' \,.\, N \stackrel{\lambda}{\Longrightarrow} N' \wedge N' \in [\![A]\!]\}$$

We have that satisfaction is closed under structural congruence.

Lemma 3.21 *We have $\equiv \subseteq =_{\mathcal{L}_w}$.*

Proof. By induction on the structure of the formulas. ∎

We now prove both inclusions for our main full abstraction result.

Lemma 3.22 *We have $\approx \subseteq =_{\mathcal{L}_w}$.*

Proof. Follows from a standard induction on the structure of the formulas. ∎

Lemma 3.23 *We have $=_{\mathcal{L}_w} \subseteq \approx$.*

Proof. Analogous to Lemma 2.19. ∎

By Lemma 3.22 and Lemma 3.23, we conclude

Theorem 3.24 (Logical Characterization of $\approx$) *We have $\approx = =_{\mathcal{L}_w}$.*

Corollary 3.25 (Logical Characterization of $\approxeq$) *We have $\approxeq = =_{\mathcal{L}_w}$.*

By Corollary 3.25 the separation power of $\mathcal{L}_w$ precisely coincides with weak reduction barbed congruence, even if it includes the spatial operators composition and void. At this point, we may ask, as at the end of Section 2.2, whether the spatial operators are essential to the characterization. We may verify that $\mathbf{T}$ can be expressed as $\langle\!\langle \tau \rangle\!\rangle \mathbf{0}$, and $\langle\!\langle \tau \rangle\!\rangle A$ as $A \Uparrow \mathbf{0}$. Thus let $\mathcal{L}_w^{min}$ be the ($\mathbf{T}$, $\langle\!\langle \tau \rangle\!\rangle A$)-free fragment of $\mathcal{L}_w$. We may show that $\mathcal{L}_w^{min}$ is as expressive as $\mathcal{L}_w$, and moreover that all of its connectives are essential for its expressiveness.

Theorem 3.26 (Minimality) *The logic $\mathcal{L}_w^{min}$ is minimal. Moreover, the operators composition and void are essential to characterize weak barbed congruence.*

Proof. (Sketch, full proof in appendix) We show that any logic obtained from $\mathcal{L}_w^{min}$ by removing each connective is strictly less expressive.

- ($\neg A$) We show that in the $\neg$-free fragment, for any N such that $N \not\equiv \mathbf{0}$ we have $N \models A$ if and only if $N \mid [\mathbf{nil}] \models A$. Hence we are not able to express property $1 \triangleq \{N \mid \exists P \,.\, N \equiv [P]\}$, nor distinguish $[\mathbf{nil}] \mid [\mathbf{nil}]$ from $[\mathbf{nil}]$.

- ($A \wedge B$) We show that in the $\wedge$-free fragment, if $N \mid [\mathbf{nil}] \models A$ then either $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models A$ or $N \models A$. Hence we cannot express property 1.

- ($\mathbf{0}$) We show that in the $\mathbf{0}$-free fragment we have that $\mathbf{0} \models A$ if and only if $[\mathbf{nil}] \models A$. Hence we can no longer express property $\{N \mid N \equiv \mathbf{0}\}$, nor tell $\mathbf{0}$ and $[\mathbf{nil}]$ apart (while $\mathbf{0} \not\cong [\mathbf{nil}]$).

- ($A \upharpoonleft\!\upharpoonright B$) We show that in the $\upharpoonleft\!\upharpoonright$ -free fragment we have that $[\mathbf{nil}] \models A$ if and only if $[\mathbf{nil}] \mid [\mathbf{nil}] \models A$. Hence we can neither express property $2 \triangleq \{N \mid \exists P, Q \,.\, N \equiv [P] \mid [Q]\}$ nor separate $[\mathbf{nil}] \mid [\mathbf{nil}]$ from $[\mathbf{nil}]$ (while $[\mathbf{nil}] \mid [\mathbf{nil}] \not\cong [\mathbf{nil}]$).

- ($\langle\!\langle \alpha \rangle\!\rangle A$, $\alpha = \bar{a}, a$) The $\langle\!\langle \alpha \rangle\!\rangle$-free fragment does not separate $[\alpha.\mathbf{nil}]$ and $[\mathbf{nil}]$.

- ($\langle\!\langle [a] \rangle\!\rangle A$) The $\langle\!\langle [a] \rangle\!\rangle$-free fragment does not separate $[\mathbf{go}.b.\mathbf{nil}]$ and $[\mathbf{nil}]$.

■

4 Concluding Remarks

We have studied observational equivalences in a distributed computation model, having obtained spatial logic characterizations of observational congruence in both the strong and weak cases. Taking as reference semantics for observational congruence the standard reduction barbed congruence, we have derived equivalent characterizations of observational congruences in terms of co-inductively defined bisimilarities. The logics considered are natural extensions of HML with spatial operators, interpreted in the standard way. We have thus shown, in a precise sense, that spatial logics, in particular the structural operators they offer, are not necessarily intensional, and may offer adequate expressive power for logically characterizing distributed behavior. We have also concluded, in the case of the specific process model here considered, that the composition operator $A \mid B$ is essential to capture (extensional) observational equivalence. Intuitively, such structural observations do not violate extensionality because distributed process behavior already has a related observational power, due to migration behavior and failures.

Observational equivalences of distributed systems have been studied extensively in the context of CCS-like models; a comprehensive survey may be found in [9]. However, it seems that logical characterizations have not been much discussed, and the distributed process equivalences proposed were technically defined by means of location or history-sensitive transition systems, where the use of location names plays a key role, both in the dynamic and static cases. Here, we build on a more abstract notion of spatial observation, avoiding the use of location names, and consider a calculus with anonymous sites and migration primitives, in the spirit of more recent proposals of calculi for distribution and mobility [7, 17].

Our adoption of the simplest fail-stop failure model was motivated by the belief that it already captures the key consequences of failure, cf., the folklore slogan that in a distributed system one cannot distinguish a failed system from a system that will respond (much) later. The fail-stop model has been frequently adopted in formalizations of failure since [1], even if recent related works prefer to trigger failure by means of an explicit "kill" primitive [11]. Failures play an essential role in our results. However, it is conceivable that other notions of failure, and a different set of spatial behaviors and spatial observations, may lead to results comparable to the ones reported in this paper.

It is interesting to compare our results with those of [13], where an extensional spatial logic (for the π-calculus) is considered. In that work, extensionality is obtained by removing the composition and void operators, while retaining the guarantee, whereas here we obtain extensionality by retaining the composition and void operators, while doing without the guarantee. We believe that the guarantee could be added to our developments, without breaking the results. Then, it would be instructive to see how to capture indirectly the action modalities, as in [14]. It would be certainly important to assess how to extend the general approach presented here to richer models, with name restriction, name passing, and full computational power.

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A Proofs of section 1 and section 2

A.1 Proofs of strong reduction barbed congruence properties

Proof of Lemma 1.8

Let P^i and Q^i $(i \in J)$ be collections of processes. We prove that if for all $i \in J$ we have $\left[P^i\right] \simeq \left[Q^i\right]$, then also $\prod_{j\in J}\left[P^j\right] \simeq \prod_{j\in J}\left[Q^j\right]$.
Proof. By induction on $\#J$. Trivial for cases $\#J = 0$ and $\#J = 1$. For $\#J = k+1$ by induction hypothesis on $J\backslash\{1\}$ we obtain that $\prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \prod_{j\in J\backslash\{1\}}\left[Q^j\right]$. Since $\simeq$ is context closed from $\left[P^1\right] \simeq \left[Q^1\right]$ we get that $\left[P^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right]$ and from $\prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \prod_{j\in J\backslash\{1\}}\left[Q^j\right]$ we conclude that $\left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[Q^j\right]$ and hence, noting that we have $\left[P^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[Q^j\right]$, we obtain $\left[P^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[P^j\right] \simeq \left[Q^1\right] \mid \prod_{j\in J\backslash\{1\}}\left[Q^j\right]$. ■

Proof of Lemma 1.9

Let M, N be networks such that $N \triangleq \prod_{j\in J}\left[P^j\right]$, where P^j $(j \in J)$ is a collection of processes, and $N \simeq M$. We prove that there is a collection of processes Q^j $(j \in J)$ such that $M \equiv \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$ we have $\left[P^j\right] \simeq \left[Q^j\right]$.
Proof. Let us consider context

$$C\left[\bullet\right] \triangleq \left[t.\mathbf{nil} \mid \textstyle\prod_{i\in J}\mathbf{go}.(f_i.\mathbf{nil} \mid \prod_{k\in J\backslash\{i\}}\bar{f}_k.fail.\mathbf{nil})\right] \mid \bullet$$

with $(\{t, fail\} \cup \{f_i \mid i \in J\}) \cap \mathit{fn}(N \mid M) = \emptyset$ and $(\{t, fail\} \cup \{f_i \mid i \in J\})$ pairwise distinct. We can derive that

$$C\left[\textstyle\prod_{j\in J}\left[P^j\right]\right] \rightarrow^{\#J+1} \prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\backslash\{j\}}\bar{f}_k.fail.\mathbf{nil}\right]$$

where $fail$ will never be observed as a barb and t is no longer exhibited. Since $\prod_{j\in J}\left[P^j\right] \simeq M$ we have that $C\left[M\right] \rightarrow^{\#J+1} M'$ and

$$\textstyle\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\backslash\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \simeq M'$$

and hence for all $i \in J$ it is the case that $M'\downarrow_{f_i}$, and also $fail$ will never be exhibited and t is no longer observed. This can only be so, attending to the fact that we have exactly $\#J+1$ reductions and since we know that they are due to the migration of the processes containing the f_is and to the failure of the site containing t, if there is $\bar{M}$ and $\{Q^j \mid j \in J\}$ such that

$$M' \equiv \textstyle\prod_{j\in J}\left[Q^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\backslash\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \mid \bar{M}$$

Since $\simeq$ is symmetric we have that $\bar{M} \equiv \mathbf{0}$ since otherwise following the same reasoning we would get a contradiction to our initial condition that in $\prod_{j\in J}\left[P^j\right]$ there are $\#J$ sites. We can at this point conclude that $M \equiv \prod_{j\in J}\left[Q^j\right]$.

We know that for all $m \in J$ we can derive

$$\textstyle\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\backslash\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \rightarrow \left[P^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\backslash\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]$$

and since $\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\backslash\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \simeq M'$ we get that there exists M_m such that $M' \rightarrow M_m$ and $\left[P^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\backslash\{m\}}\bar{f}_k.fail.\mathbf{nil}\right] \simeq M_m$, which, since $\simeq$ identifies systems with the same number of sites as we proved before and recalling that it is barb closed, gives us that $M_m \equiv \left[Q^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\backslash\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]$.

We now consider context

$$C\left[\bullet\right] \triangleq \left[r.\mathbf{nil} \mid \mathbf{go}.(\bar{f}_m.\mathbf{nil} \mid \textstyle\prod_{k\in J\backslash\{m\}}f_k.\bar{fail}.\mathbf{nil})\right] \mid \bullet$$

with $r \notin \{t, fail\} \cup \{f_i \mid i \in J\} \cup \mathit{fn}(N \mid M)$. We can derive that

$$C\left[\left[P^m \mid f_m.\mathbf{nil} \mid \textstyle\prod_{k\in J\backslash\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]\right] \rightarrow^{2\times\#J} \left[P^m\right] \mid \left[r.\mathbf{nil}\right] \rightarrow \left[P^m\right]$$

which gives us that

$C\left[\left[Q^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]\right] \rightarrow^{(2\times\#J)+1} [Q^m]$

and $[P^m] \simeq [Q^m]$ thus completing the proof. ■

A.2 Proofs of strong bisimilarity properties

Proof of Lemma 2.9

Let M be a network and $P^j (j \in J)$ a collection of processes such that $\prod_{j\in J}\left[P^j\right] \sim M$. We prove there is a collection of processes $Q^j(j \in J)$ such that $M \equiv \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$, $\left[P^j\right] \sim \left[Q^j\right]$.

Proof. By induction on $\#J$. Trivial for case $\#J = 0$. For $\#J = 1$ aiming at a contradiction let us assume that there is no Q such that $M \equiv [Q]$ which can only be so if either $M \equiv \mathbf{0}$ or there exist Q_1, Q_2, M' such that $M \equiv [Q_1] \mid [Q_2] \mid M'$. Let us first consider $M \equiv \mathbf{0}$ from which, since $M \sim \left[P^1\right]$, we obtain that $\left[P^1\right] \equiv \mathbf{0}$, which gives us our intended contradiction. Let us now consider that there exist Q_1, Q_2, M' such that $M \equiv [Q_1] \mid [Q_2] \mid M'$ which since $M \sim \left[P^1\right]$ gives us that there exist N_1, N_2 such that $\left[P^1\right] \equiv N_1 \mid N_2$ and $[Q_1] \sim N_1$ and $[Q_2] \mid M' \sim N_2$. Since $\left[P^1\right] \equiv N_1 \mid N_2$ we have that either $N_1 \equiv \mathbf{0}$ or $N_2 \equiv \mathbf{0}$ which leads to a contradiction since $N_1 \sim [Q_1]$ and $N_2 \sim [Q_2] \mid M'$ gives us that $[Q_1] \equiv \mathbf{0}$ or $[Q_2] \mid M' \equiv \mathbf{0}$. Thus there is Q such that $M \equiv [Q]$.

For $\#J = k + 1$ from $\prod_{j\in J}\left[P^j\right] \equiv \left[P^1\right] \mid \prod_{j\in J\setminus\{1\}}\left[P^j\right]$ and $\prod_{j\in J}\left[P^j\right] \sim M$ we get that there exist M_1, M_2 such that $M \equiv M_1 \mid M_2$ and $\left[P^1\right] \sim M_1$ and $\prod_{j\in J\setminus\{1\}}\left[P^j\right] \sim M_2$. From $\left[P^1\right] \sim M_1$ we have that there exists Q^1 such that $M_1 \equiv \left[Q^1\right]$ and by induction hypothesis on $\prod_{j\in J\setminus\{1\}}\left[P^j\right] \sim M_2$ we have that there exists $\{Q^j \mid j \in J\setminus\{1\}\}$ such that $M_2 \equiv \prod_{j\in J\setminus\{1\}}\left[Q^j\right]$ and for all $j \in J\setminus\{1\}$ it is the case that $\left[P^j\right] \sim \left[Q^j\right]$ which completes the proof. ■

Proof of Lemma 2.10

Let J be a finite set and I_j, for all $j \in J$, be a finite set. Let P_i^j and Q_i^j be processes such that for all $j \in J$ and $i \in I_j$ we have $\left[P_i^j\right] \sim \left[Q_i^j\right]$. We prove that $\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right] \sim \prod_{j\in J}\left[\prod_{i\in I_j} Q_i^j\right]$.

Proof. We abbreviate $\prod_{j\in J}\left[\prod_{i\in I_j} P_i^j\right]$ and $\prod_{j\in J}\left[\prod_{i\in I_j} Q_i^j\right]$ with N^J and M^J, respectively, and proceed by coinduction on the definition of strong bisimulation.

Let us consider that there exist N', N'' such that $N^J \equiv N' \mid N''$. We know that there exists $\bar{J} \subseteq J$ such that $N^{\bar{J}} \equiv N'$ and $N^{J\setminus\bar{J}} \equiv N''$. We also know that $M^J \equiv M^{\bar{J}} \mid M^{J\setminus\bar{J}}$. Since $\bar{J} \subseteq J$ we have that for all $j \in \bar{J}$ and $i \in I_j$ it is the case that $\left[P_i^j\right] \sim \left[Q_i^j\right]$ which gives us that $(N^{\bar{J}}, M^{\bar{J}}) \in B$ and also since for all $j \in J\setminus\bar{J}$ and $i \in I_j$ it is the case that $\left[P_i^j\right] \sim \left[Q_i^j\right]$ from which we get that $(N^{J\setminus\bar{J}}, M^{J\setminus\bar{J}}) \in B$, thus proving the first clause.

Now consider that $N^J \equiv \mathbf{0}$ which gives us that $\#J = 0$ and hence we directly have that $M^J \equiv \mathbf{0}$, thus proving the second clause.

Let us now consider that there exists λ, N' such that $N^J \xrightarrow{\lambda} N'$. We know that this transition can either be triggered by an unique site or else be a migration of a process from one site to another or else be due to a grow transition or finally be due to a failure.

(*Transition triggered by a single site*)

If a site triggers the transition, this can be due either to a firing of an action that can either be an input or an output or an internal action, due to either a synchronization between processes internal to one of the P_i^js or to a τ prefix, or else to a synchronization between two distinct P_i^js. Hence we can write that there exist $\bar{j} \in J$ and $n, m \in I_{\bar{j}}$ and $\bar{P}^{\bar{j}}_n, \bar{P}^{\bar{j}}_m$ such that either $\left[P^{\bar{j}}_n\right] \xrightarrow{\lambda} \left[\bar{P}^{\bar{j}}_n\right]$, with $\lambda = \alpha$, or $\left[P^{\bar{j}}_n \mid P^{\bar{j}}_m\right] \xrightarrow{\tau} \left[\bar{P}^{\bar{j}}_n \mid \bar{P}^{\bar{j}}_m\right]$.

(*case of* $N^J \xrightarrow{\lambda} N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n\}} P^{\bar{j}}_i \mid \bar{P}^{\bar{j}}_n\right], \lambda = \alpha$)

We know that $\left[P^{\bar{j}}_n\right] \xrightarrow{\lambda} \left[\bar{P}^{\bar{j}}_n\right]$, which since $\left[P^{\bar{j}}_n\right] \sim \left[Q^{\bar{j}}_n\right]$ gives us that there exists M' such that $\left[Q^{\bar{j}}_n\right] \xrightarrow{\lambda} M'$ and $\left[\bar{P}^{\bar{j}}_n\right] \sim M'$ which, considering Lemma 2.9, leads to there exists $\bar{Q}^{\bar{j}}_n$ such that $M' \equiv \left[\bar{Q}^{\bar{j}}_n\right]$.

Hence we can derive $M^J \xrightarrow{\lambda} M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n\}} Q^{\bar{j}}_i \mid \bar{Q}^{\bar{j}}_n\right]$, which along with
$(N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n\}} P^{\bar{j}}_i \mid \bar{P}^{\bar{j}}_n\right], M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n\}} Q^{\bar{j}}_i \mid \bar{Q}^{\bar{j}}_n\right]) \in B$
since $\left[\bar{P}^{\bar{j}}_n\right] \sim \left[\bar{Q}^{\bar{j}}_n\right]$, completes the proof for this case.

(*case of* $N^J \xrightarrow{\tau} N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n,m\}} P^{\bar{j}}_i \mid \bar{P}^{\bar{j}}_n \mid \bar{P}^{\bar{j}}_m\right]$)

Since a synchronization can take place we know that there exists a such that either $P^{\bar{j}}_n \equiv_p \bar{a}.R_1 \mid R_2$ and $P^{\bar{j}}_m \equiv_p a.R_3 \mid R_4$ or with the action and coaction placed the other way around, being the proofs analogous. Considering $P^{\bar{j}}_n \equiv_p \bar{a}.R_1 \mid R_2$ and $P^{\bar{j}}_m \equiv_p a.R_3 \mid R_4$ we have that $\left[P^{\bar{j}}_n\right] \xrightarrow{\bar{a}} [R_1 \mid R_2]$ and $\left[P^{\bar{j}}_m\right] \xrightarrow{a} [R_3 \mid R_4]$ being that $[R_1 \mid R_2] \equiv \left[\bar{P}^{\bar{j}}_n\right]$ and $[R_3 \mid R_4] \equiv \left[\bar{P}^{\bar{j}}_m\right]$. Since $\left[P^{\bar{j}}_n\right] \sim \left[Q^{\bar{j}}_n\right]$ we obtain that $\left[Q^{\bar{j}}_n\right] \xrightarrow{\bar{a}} M'$ and $\left[\bar{P}^{\bar{j}}_n\right] \sim M'$ which, considering Lemma 2.9, leads to there exists $\bar{Q}^{\bar{j}}_n$ such that $M' \equiv \left[\bar{Q}^{\bar{j}}_n\right]$. Also since $\left[P^{\bar{j}}_m\right] \sim \left[Q^{\bar{j}}_m\right]$ we get that $\left[Q^{\bar{j}}_m\right] \xrightarrow{a} M''$ and $\left[\bar{P}^{\bar{j}}_m\right] \sim M''$ from which, considering Lemma 2.9, we obtain that there exists $\bar{Q}^{\bar{j}}_m$ such that $M'' \equiv \left[\bar{Q}^{\bar{j}}_n\right]$. Hence we can derive that $M^J \xrightarrow{\tau} M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n,m\}} Q^{\bar{j}}_i \mid \bar{Q}^{\bar{j}}_n \mid \bar{Q}^{\bar{j}}_m\right]$, which along with
$(N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n,m\}} P^{\bar{j}}_i \mid \bar{P}^{\bar{j}}_n \mid \bar{P}^{\bar{j}}_m\right], M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{n,m\}} Q^{\bar{j}}_i \mid \bar{Q}^{\bar{j}}_n \mid \bar{Q}^{\bar{j}}_m\right]) \in B$
since $\left[\bar{P}^{\bar{j}}_n\right] \sim \left[\bar{Q}^{\bar{j}}_n\right]$ and $\left[\bar{P}^{\bar{j}}_m\right] \sim \left[\bar{Q}^{\bar{j}}_m\right]$, completes the proof for this case.

(*Transition triggered by a migration*)

We now consider that a migration takes place, for which we know that there are at least two sites involved, the origin and destination of the migrating process, hence we have that $\#J \geq 2$. We also know that there exist $\bar{j}, \bar{i}$ such that $P^{\bar{j}}_{\bar{i}} \equiv_p \mathbf{go}.R_2 \mid R_1$ from which we can obtain, considering $a \notin \mathit{fn}(\left[P^{\bar{j}}_{\bar{i}}\right] \mid \left[Q^{\bar{j}}_{\bar{i}}\right])$, that $\left[P^{\bar{j}}_{\bar{i}}\right] \xrightarrow{[a]} \left[P^{\bar{j}}_{\bar{i}}\right] \mid [a.\mathbf{nil}] \xrightarrow{\tau} [R_1] \mid [a.\mathbf{nil} \mid R_2]$. Since $\left[P^{\bar{j}}_{\bar{i}}\right] \sim \left[Q^{\bar{j}}_{\bar{i}}\right]$ we get that there exists M' such that $\left[Q^{\bar{j}}_{\bar{i}}\right] \xrightarrow{[a]} \left[Q^{\bar{j}}_{\bar{i}}\right] \mid [a.\mathbf{nil}] \xrightarrow{\tau} M'$ and $[R_1] \mid [a.\mathbf{nil} \mid R_2] \sim M'$, from which we can derive that there exist M'_1, M'_2 such that $M' \equiv M'_1 \mid M'_2$ and $[R_1] \sim M'_1$ and $[a.\mathbf{nil} \mid R_2] \sim M'_2$ which, noting that $a \notin \mathit{fn}(\left[Q^{\bar{j}}_{\bar{i}}\right])$ and considering Lemma 2.9, leads to there exist R_3, R_4 such that $M'_1 \equiv [R_3]$ and $M'_2 \equiv [a.\mathbf{nil} \mid R_4]$. From $[a.\mathbf{nil} \mid R_2] \sim [a.\mathbf{nil} \mid R_4]$ and $[a.\mathbf{nil} \mid R_2] \xrightarrow{a} [R_2]$ we get that $[a.\mathbf{nil} \mid R_4] \xrightarrow{a} [R_4]$ and $[R_2] \sim [R_4]$.

So we have that there exists $\bar{l} \in J$ such that
$N^J \xrightarrow{\tau} N^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{\bar{i}\}} P^{\bar{j}}_i \mid R_1\right] \mid \left[\prod_{i \in I_{\bar{l}}} P^{\bar{l}}_i \mid R_2\right]$.

Also from $\left[Q^{\bar{j}}_{\bar{i}}\right] \xrightarrow{[a]} \left[Q^{\bar{j}}_{\bar{i}}\right] \mid [a.\mathbf{nil}] \xrightarrow{\tau} [R_3] \mid [a.\mathbf{nil} \mid R_4]$ we can derive that
$M^J \xrightarrow{\tau} M^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i \in I_{\bar{j}}\setminus\{\bar{i}\}} Q^{\bar{j}}_i \mid R_3\right] \mid \left[\prod_{i \in I_{\bar{l}}} Q^{\bar{l}}_i \mid R_4\right]$,

which along with

$$(\quad N^{J\backslash\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\backslash\{\bar{i}\}} P_i^{\bar{j}} \mid R_1\right] \mid \left[\prod_{i\in I_{\bar{l}}} P_i^{\bar{l}} \mid R_2\right],$$
$$M^{J\backslash\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\backslash\{\bar{i}\}} Q_i^{\bar{j}} \mid R_3\right] \mid \left[\prod_{i\in I_{\bar{l}}} Q_i^{\bar{l}} \mid R_4\right]) \in B$$

since $[R_1] \sim [R_3]$ and $[R_2] \sim [R_4]$, completes the proof for this case.

(*Transition triggered by a grow transition*)

We have that $N^J \xrightarrow{[a]} N^J \mid [a.\mathbf{nil}]$ and we can directly derive that $M^J \xrightarrow{[a]} M^J \mid [a.\mathbf{nil}]$ which along with $(N^J \mid [a.\mathbf{nil}], M^J \mid [a.\mathbf{nil}]) \in B$, since $[a.\mathbf{nil}] \sim [a.\mathbf{nil}]$, completes the proof for this case.

(*Transition triggered by a failure*)

We have that there exists $\bar{J} \subseteq J$ such that $N^J \xrightarrow{\tau} N^{J\backslash\bar{J}}$. We can directly derive that $M^J \xrightarrow{\tau} M^{J\backslash\bar{J}}$ which along with $(N^{J\backslash\bar{J}}, M^{J\backslash\bar{J}}) \in B$ completes the proof for this case and also for this clause. ■

Proof of Lemma 2.11

We prove strong bisimilarity is a congruence.

Proof. We know that there exist J and $\{P^j \mid j \in J\}$ such that $N \equiv \prod_{j\in J}[P^j]$ which considering that $N \sim M$ and Lemma 2.9 gives us that there exists $\{Q^j \mid j \in J\}$ such that $M \equiv \prod_{j\in J}[Q^j]$ and for all $j \in J$ we have that $[P^j] \sim [Q^j]$. Also we know that there exists C such that $C[N] \equiv N \mid C$ and $C[M] \equiv M \mid C$ which, along with the fact that there exist I and $\{R^i \mid i \in I\}$ such that $C \equiv \prod_{i\in I}[R^i]$, gives us that $C[N] \equiv \prod_{j\in J}[P^j] \mid \prod_{i\in I}[R^i]$ and $C[M] \equiv \prod_{j\in J}[Q^j] \mid \prod_{i\in I}[R^i]$, which along with for all $j \in J$ it is the case that $[P^j] \sim [Q^j]$ and for all $i \in I$ it is the case that $[R^i] \sim [R^i]$ and considering Lemma 2.10 gives us that $C[N] \sim C[M]$. ■

A.3 Proofs of full abstraction ($\sim = \simeq$)

Proof of Lemma 2.12

We prove $\sim \subseteq \simeq$.

Proof. We proceed by coinduction on the definition of reduction barbed congruence. Let us consider N, M such that $N \sim M$.

Consider now that there exists a such that $N\downarrow_a$. This means that there exist P_1, P_2, N' such that $N \equiv [a.P_1 \mid P_2] \mid N'$ from which we can derive $N \xrightarrow{a} [P_1 \mid P_2] \mid N'$ and hence, since $N \sim M$, we have that there exists $\bar{M}$ such that $M \xrightarrow{a} \bar{M}$ and $[P_1 \mid P_2] \mid N' \sim \bar{M}$. This gives us that there exist Q_1, Q_2, M' such that $M \equiv [a.Q_1 \mid Q_2] \mid M'$ hence $M\downarrow_a$, and completes the proof for the first clause.

Let us now consider that there exists N' such that $N \to N'$. We can derive that $N \xrightarrow{\tau} N'$ which, since $N \sim M$, gives us that there exists M' such that $M \xrightarrow{\tau} M'$ and $N' \sim M'$. From $M \xrightarrow{\tau} M'$ we get that $M \to M'$ which along with $N' \sim M'$ completes the proof of the second clause.

Lemma 2.11 directly gives us $C[N] \sim C[M]$ thus proving the third clause. ■

Proof of Lemma 2.13

We prove $\simeq \subseteq \sim$.

Proof. We proceed by coinduction on the definition of strong bisimulation. Let us consider N, M such that $N \simeq M$.

Consider now that there exist N', N'' such that $N \equiv N' \mid N''$. We know that there exist J and $\{P^j \mid j \in J\}$ such that $N \equiv \prod_{j\in J}[P^j]$ and also that there exists $\bar{J} \subseteq J$ such that $N' \equiv \prod_{j\in \bar{J}}[P^j]$ and $N'' \equiv \prod_{j\in J\setminus\bar{J}}[P^j]$. From $N \equiv \prod_{j\in J}[P^j]$ and $N \simeq M$, considering Lemma 1.9, we have that there exists $\{Q^j \mid j \in J\}$ such that $M \equiv \prod_{j\in J}[Q^j]$ and for all $j \in J$ it is the case that $[P^j] \simeq [Q^j]$. We can now write that $M \equiv \prod_{j\in\bar{J}}[Q^j] \mid \prod_{j\in J\setminus\bar{J}}[Q^j]$. From the fact that for all $j \in \bar{J}$ it is the case that $[P^j] \simeq [Q^j]$ and for all $j \in J\setminus\bar{J}$ it is the case that $[P^j] \simeq [Q^j]$, considering Lemma 1.8, we obtain $\prod_{j\in\bar{J}}[P^j] \simeq \prod_{j\in\bar{J}}[Q^j]$ and $\prod_{j\in J\setminus\bar{J}}[P^j] \simeq \prod_{j\in J\setminus\bar{J}}[Q^j]$, which completes the proof for this case.

Now consider that $N \equiv \mathbf{0}$. Lemma 1.9 provides directly that $M \equiv \mathbf{0}$.

Consider now that there exist λ, N' such that $N \xrightarrow{\lambda} N'$. We have that λ is either τ, or else there exists a such that $\lambda = \bar{a}$ or $\lambda = a$ or finally that there exists a such that $\lambda = [a]$. If $\lambda = \tau$ we have that $N \to N'$ and since $N \simeq M$ we get that there exists M' such that $M \to M'$, hence $M \xrightarrow{\tau} M'$, and $N' \simeq M'$ which completes the proof for this case.

If there exists a such that $\lambda = \bar{a}$ we have that there exist P_1, P_2, N'' such that $N \equiv [\bar{a}.P_1 \mid P_2] \mid N''$ and $N' \equiv [P_1 \mid P_2] \mid N''$. Let us consider context

$C[\bullet] \triangleq [t.\mathbf{nil} \mid \mathbf{go}.a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil})] \mid \bullet$

with $t, f \notin \mathit{fn}(N \mid M)$ and $t \neq f$. We can easily derive that

$C[N] \to [a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil}) \mid \bar{a}.P_1 \mid P_2] \mid [t.\mathbf{nil}] \mid N'' \to$
$[a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil}) \mid \bar{a}.P_1 \mid P_2] \mid N'' \to [f.\mathbf{nil} \mid \bar{f}.\mathbf{nil} \mid P_1 \mid P_2] \mid N'' \to [P_1 \mid P_2] \mid N''$

which, since $N \simeq M$, gives us that $C[M] \to^4 M'$ and $[P_1 \mid P_2] \mid N'' \simeq M'$. Since barb f is observed only after three steps, and recalling that $f \notin \mathit{fn}(M)$, and barb t is observed only up to one step we obtain that there exist Q_1, Q_2, M'' such that $M \equiv [\bar{a}.Q_1 \mid Q_2] \mid M''$ and $M' \equiv [Q_1 \mid Q_2] \mid M''$ which gives us that $M \xrightarrow{\bar{a}} M'$ and since $N' \simeq M'$ the proof is complete, being the proof for $\lambda = a$ analogous.

If $\lambda = [a]$ we have that there exists N' such that $N \xrightarrow{[a]} N'$ and $N' \equiv N \mid [a.\mathbf{nil}]$. Since we know N is not empty we also know that M is not empty and hence we have that $M \xrightarrow{[a]} M \mid [a.\mathbf{nil}]$ and since $\simeq$ is context closed we have $N \mid [a.\mathbf{nil}] \simeq M \mid [a.\mathbf{nil}]$, thus completing the proof. ∎

A.4 Proofs of logical characterization of $\sim$

Proof of Lemma 2.18

We prove $\sim \subseteq =_{\mathcal{L}_s}$.

Proof. We prove that if $N \sim M$ then for all A if $N \models A$ then $M \models A$ (the symmetric is analogous to prove) and we do so by induction on the structure of formula A.

If $A = \mathbf{T}$ then $M \models A$.

If $A = \neg B$ we have that $N \not\models B$. Let us supose, aiming at a contradiction, that $M \models B$ which, by induction hypothesis, gives us $N \models B$ which is a contradiction and hence $M \not\models B$ and $M \models \neg B$.

If $A = B \wedge C$ then $N \models B$ and $N \models C$ from which, by induction hypothesis, we obtain $M \models B$ and $M \models C$ hence $M \models B \wedge C$.

If $A = B \mid C$ then there exist N', N'' such that $N \equiv N' \mid N''$ and $N' \models B$ and $N'' \models C$. From $N \sim M$ we get that there exist M', M'' such that $M \equiv M' \mid M''$ and $N' \sim M'$ and $N'' \sim M''$ from which, by induction hypothesis, we obtain $M' \models B$ and $M'' \models C$ hence $M \models B \mid C$.

If $A = \langle\lambda\rangle B$ we have that there exists N' such that $N \xrightarrow{\lambda} N'$ and $N' \models B$. From $N \sim M$ we get that there exists M' such that $M \xrightarrow{\lambda} M'$ and $N' \sim M'$ which, by induction hypothesis, gives us that $M' \models B$ hence $M \models \langle\lambda\rangle B$. ■

Proof of Lemma 2.19

We prove $=_{\mathcal{L}_s} \subseteq \sim$.
Proof. We prove that $R \triangleq \{(N, M) \mid N =_{\mathcal{L}_s} M\}$ is a strong bisimulation by coinduction on the definition of strong bisimulation.

Let us consider that there exist N', M' such that $N \equiv N' \mid N''$. Let us consider $I \triangleq \{1, 2, \ldots, k\}$ and $\{M'_i, M''_i \mid i \in I\}$ such that for all M', M'' such that $M \equiv M' \mid M''$ then there exists $i \in I$ such that $M' \equiv M'_i$ and $M'' \equiv M''_i$. Aiming at a contradiction, let us now assume that for all $i \in I$ it is either the case that $N' \neq_{\mathcal{L}_s} M'_i$ or $N'' \neq_{\mathcal{L}_s} M''_i$ from which we can derive that there exists $\{A_i, B_i \mid i \in I\}$ such that for all $i \in I$ it is either the case that $N' \models A_i$ and $M'_i \not\models A_i$ or $N'' \models B_i$ and $M''_i \not\models B_i$. We can now write that $N \models (\bigwedge_{i \in I} A_i) \mid (\bigwedge_{i \in I} B_i)$ and since $N =_{\mathcal{L}_s} M$ we have that $M \models (\bigwedge_{i \in I} A_i) \mid (\bigwedge_{i \in I} B_i)$ which gives us that there exist M', M'' such that $M \equiv M' \mid M''$ and $M' \models (\bigwedge_{i \in I} A_i)$ and $M'' \models (\bigwedge_{i \in I} B_i)$. We also know that there exists $j \in I$ such that $M' \equiv M'_j$ and $M'' \equiv M''_j$ from which follows that $M'_j \models (\bigwedge_{i \in I} A_i)$ and $M''_j \models (\bigwedge_{i \in I} B_i)$ which provides with the intended contradiction since $M'_j \not\models A_j$ or $M''_j \not\models B_j$. We can therefore conclude that there exists $i \in I$ such that $N' =_{\mathcal{L}_s} M'_i$ and $N'' =_{\mathcal{L}_s} M''_i$, which completes the proof.

Now let us consider that $N \equiv \mathbf{0}$. We know that $N \models \mathbf{0}$ from which, since $N =_{\mathcal{L}_s} M$, we get that $M \models \mathbf{0}$ hence $M \equiv \mathbf{0}$.

Let us now consider that there exist N', λ such that $N \xrightarrow{\lambda} N'$, hence $N \models \langle\lambda\rangle\mathbf{T}$ which since $N =_{\mathcal{L}_s} M$ gives us that $M \models \langle\lambda\rangle\mathbf{T}$ and thus there exists M' such that $M \xrightarrow{\lambda} M'$. Let us consider $I \triangleq \{1, 2, \ldots, k\}$ and $\{M'_i \mid i \in I\}$ such that for all M' such that $M \xrightarrow{\lambda} M'$ then there exists $i \in I$ such that $M' \equiv M'_i$. Aiming at a contradiction, let us now assume that for all $i \in I$ it is the case that $N' \neq_{\mathcal{L}_s} M'_i$ which gives us that there exists $\{A_i \mid i \in I\}$ such that for all $i \in I$ it is the case that $N' \models A_i$ and $M'_i \not\models A_i$. We can now write that $N \models \langle\lambda\rangle(\bigwedge_{i \in I} A_i)$ and since $N =_{\mathcal{L}_s} M$ we have $M \models \langle\lambda\rangle(\bigwedge_{i \in I} A_i)$ from which we obtain that there exists M' such that $M \xrightarrow{\lambda} M'$ and $M' \models \bigwedge_{i \in I} A_i$. We also know that there exists $j \in I$ such that $M' \equiv M'_j$ which gives us $M'_j \models \bigwedge_{i \in I} A_i$ which contradicts $M'_j \not\models A_j$, hence there exists $i \in I$ such that $N' =_{\mathcal{L}_s} M'_i$ which completes the proof. ■

B Proofs of section 3

B.1 Proofs of weak reduction barbed congruence properties

Proof of Lemma 3.5

Let P^i and Q^i $(i \in J)$ be collections of processes. We prove that if for all $i \in J$ we have $\left[P^i\right] \approx \left[Q^i\right]$, then also $\prod_{j \in J}\left[P^j\right] \approx \prod_{j \in J}\left[Q^j\right]$.
Proof. By induction on J. Trivial for cases $\#J = 0$ and $\#J = 1$. By induction hypothesis on $J \backslash \{1\}$ we obtain that $\prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \prod_{j \in J\backslash\{1\}}\left[Q^j\right]$. Since $\approx$ is context closed from $\left[P^1\right] \approx \left[Q^1\right]$ we have that $\left[P^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right]$ and from $\prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \prod_{j \in J\backslash\{1\}}\left[Q^j\right]$ we conclude that $\left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[Q^j\right]$ and hence, noting that we have $\left[P^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[Q^j\right]$, we obtain $\left[P^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[P^j\right] \approx \left[Q^1\right] \mid \prod_{j \in J\backslash\{1\}}\left[Q^j\right]$. ■

Proof of Lemma 3.6

Let M, N be networks such that $N \triangleq \prod_{j\in J}\left[P^j\right]$, where P^j $(j \in J)$ is a collection of processes, and $N \approxeq M$. We prove that there is a collection of processes Q^j $(j \in J)$ such that $M \Rightarrow \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$ we have $\left[P^j\right] \approxeq \left[Q^j\right]$.

Proof. Let us consider context

$$C\,[\bullet] \triangleq \left[t.\mathbf{nil} \mid \prod_{i\in J}\mathbf{go}.(f_i.\mathbf{nil} \mid \prod_{k\in J\setminus\{i\}}\bar{f}_k.fail.\mathbf{nil})\right] \mid \bullet$$

with $(\{t, fail\} \cup \{f_i \mid i \in J\}) \cap fn(N \mid M) = \emptyset$ and $(\{t, fail\} \cup \{f_i \mid i \in J\})$ pairwise distinct. We can derive that

$$C\left[\prod_{j\in J}\left[P^j\right]\right] \rightarrow^{\#J+1} \prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\setminus\{j\}}\bar{f}_k.fail.\mathbf{nil}\right]$$

where $fail$ will never be observed as a barb and t is no longer exhibited. Since $\prod_{j\in J}\left[P^j\right] \approxeq M$ we have that $C\,[M] \Rightarrow^{\#J+1} M'$ and

$$\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\setminus\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \approxeq M'$$

and hence for all $i \in J$ it is the case that $M'\Downarrow_{f_i}$, and also $fail$ will never be exhibited and t is no longer observed. This can only be so, attending to the fact that sites can not be created in a sequence of reductions and also regarding that migrations that originated from the considered context have already occurred (barbs f_i will be available) since the site has failed (t is no longer observable), if there exist $\bar{M}$ and $\{R^j \mid j \in J\}$ such that

$$M' \equiv \prod_{j\in J}\left[R^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\setminus\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \mid \bar{M}$$

Since $\approxeq$ is symmetric we have that M initially has $\#J$ sites since otherwise following the same reasoning we would get a contradiction to our initial condition that $\prod_{j\in J}\left[P^j\right]$ has $\#J$ sites, hence we can conclude that $\bar{M} \equiv \mathbf{0}$.

We know that for all $m \in J$ we can derive that

$$\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\setminus\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \rightarrow \left[P^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]$$

and since $\prod_{j\in J}\left[P^j \mid f_j.\mathbf{nil} \mid \prod_{k\in J\setminus\{j\}}\bar{f}_k.fail.\mathbf{nil}\right] \approxeq M'$ we get that there exists M_m such that $M' \Rightarrow M_m$ and $\left[P^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right] \approxeq M_m$, which, since $\approxeq$ identifies systems with the same number of sites as we proved before and recalling that it is barb closed, gives us that there exists $\bar{Q}^m$ such that $M_m \equiv \left[\bar{Q}^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]$.

We now consider context

$$C\,[\bullet] \triangleq \left[r.\mathbf{nil} \mid \mathbf{go}.(\bar{f_m}.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}f_k.\bar{fail}.\mathbf{nil})\right] \mid \bullet$$

with $r \notin \{t, fail\} \cup \{f_i \mid i \in J\} \cup fn(N \mid M)$. We can derive that

$$C\left[\left[P^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]\right] \rightarrow^{2\times\#J} [P^m] \mid [r.\mathbf{nil}] \rightarrow [P^m]$$

which gives us that there exists Q^m such that

$$C\left[\left[\bar{Q}^m \mid f_m.\mathbf{nil} \mid \prod_{k\in J\setminus\{m\}}\bar{f}_k.fail.\mathbf{nil}\right]\right] \Rightarrow^{(2\times\#J)+1} [Q^m]$$

and $[P^m] \approxeq [Q^m]$ thus completing the proof. ∎

B.2 Proofs of weak bisimilarity properties

Proof of Lemma 3.13

Let M be a network and P^j $(j \in J)$ a collection of processes such that $\prod_{j\in J}\left[P^j\right] \approx M$. We prove there is a collection of processes Q^j $(j \in J)$ such that $M \Rightarrow \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$, $\left[P^j\right] \approx \left[Q^j\right]$.

Proof. By induction on $\#J$. Trivial for case $\#J = 0$. Case $\#J = 1$ we have that $[P] \approx M$. Aiming at a contradiction let us assume that there exists no Q such that $M \equiv [Q]$ which can only be so if either $M \equiv \mathbf{0}$ or there exist Q_1, Q_2, M' such that $M \equiv [Q_1] \mid [Q_2] \mid M'$. Let us first consider $M \equiv \mathbf{0}$ from which, since $M \approx [P]$, we obtain that $[P] \equiv \mathbf{0}$, which gives us our intended contradiction. Let us now consider that there exist Q_1, Q_2, M' such that $M \equiv [Q_1] \mid [Q_2] \mid M'$ which since $M \approx [P]$ gives us that there exist N_1, N_2 such that $[P] \Rightarrow N_1 \mid N_2$ and $[Q_1] \approx N_1$ and $[Q_2] \mid M' \approx N_2$. Since $[P] \Rightarrow N_1 \mid N_2$ and attending to the fact that sites can not be created in a sequence of reductions we have that either $N_1 \equiv \mathbf{0}$ or $N_2 \equiv \mathbf{0}$ which leads to a contradiction since $N_1 \approx [Q_1]$ and $N_2 \approx [Q_2] \mid M'$ gives us that $[Q_1] \equiv \mathbf{0}$ or $[Q_2] \mid M' \equiv \mathbf{0}$. Thus there exists Q such that $M \equiv [Q]$.

From $\prod_{j \in J} \left[P^j\right] \equiv \left[P^1\right] \mid \prod_{j \in J \setminus \{1\}} \left[P^j\right]$ and $\prod_{j \in J} \left[P^j\right] \approx M$ we get that there exist M_1, M_2 such that $M \Rightarrow M_1 \mid M_2$ and $\left[P^1\right] \approx M_1$ and $\prod_{j \in J \setminus \{1\}} \left[P^j\right] \approx M_2$. From $\left[P^1\right] \approx M_1$ and considering case $\#J = 1$ we have that there exists Q^1 such that $M_1 \equiv \left[Q^1\right]$ and by induction hypothesis on $\prod_{j \in J \setminus \{1\}} \left[P^j\right] \approx M_2$ we have that there exists $\{Q^j \mid j \in J \setminus \{1\}\}$ such that $M_2 \Rightarrow \prod_{j \in J \setminus \{1\}} \left[Q^j\right]$ and for all $j \in J \setminus \{1\}$ it is the case that $\left[P^j\right] \approx \left[Q^j\right]$ which, since $M \Rightarrow \left[Q^1\right] \mid \prod_{j \in J \setminus \{1\}} \left[Q^j\right]$, completes the proof. ∎

Proof of Lemma 3.14

Let J be a finite set and I_j, for all $j \in J$, be a finite set. Let P_i^j and Q_i^j be two collections of processes such that for all $j \in J$ and $i \in I_j$ we have $\left[P_i^j\right] \approx \left[Q_i^j\right]$. We prove that $\prod_{j \in J} \left[\prod_{i \in I_j} P_i^j\right] \approx \prod_{j \in J} \left[\prod_{i \in I_j} Q_i^j\right]$.

Proof. We abbreviate $\prod_{j \in J} \left[\prod_{i \in I_j} P_i^j\right]$ and $\prod_{j \in J} \left[\prod_{i \in I_j} Q_i^j\right]$ with N^J and M^J, respectively, and proceed by coinduction on the definition of weak bisimulation.

Let us consider that there exist N', N'' such that $N^J \equiv N' \mid N''$. We know that there exists $\bar{J} \subseteq J$ such that $N^{\bar{J}} \equiv N'$ and $N^{J \setminus \bar{J}} \equiv N''$. We also know that $M^J \Rightarrow M^{\bar{J}} \mid M^{J \setminus \bar{J}}$. Since $\bar{J} \subseteq J$ we have that for all $j \in \bar{J}$ and $i \in I_j$ it is the case that $\left[P_i^j\right] \approx \left[Q_i^j\right]$ which gives us that $(N^{\bar{J}}, M^{\bar{J}}) \in B$ and also since for all $j \in J \setminus \bar{J}$ and $i \in I_j$ it is the case that $\left[P_i^j\right] \approx \left[Q_i^j\right]$ we have $(N^{J \setminus \bar{J}}, M^{J \setminus \bar{J}}) \in B$ thus proving the first clause.

Now consider that $N^J \equiv \mathbf{0}$, which gives us that $\#J = 0$ and hence we directly have that $M^J \equiv \mathbf{0}$ thus proving the second clause.

Let us now consider that there exist λ and N' such that $N^J \xrightarrow{\lambda} N'$. We know that this transition can either be triggered by an unique site or else be a migration of a process from one site to another or else be due to a grow transition of finally be due to a failure.

(*Transition triggered by a single site*)

If a site triggers the transition, this can be due either to a firing of an action that can either be an input or an output or an internal action, due to either a synchronization between processes internal to one of the P_i^js or to a τ prefix, or else to a synchronization between two distinct P_i^js. Hence we can write that there exist $\bar{j} \in J$ and $n, m \in I_{\bar{j}}$ and $\bar{P}_n^{\bar{j}}, \bar{P}_m^{\bar{j}}$ such that either $\left[P_n^{\bar{j}}\right] \xrightarrow{\lambda} \left[\bar{P}_n^{\bar{j}}\right]$, with $\lambda = \alpha$, or $\left[P_n^{\bar{j}} \mid P_m^{\bar{j}}\right] \xrightarrow{\tau} \left[\bar{P}_n^{\bar{j}} \mid \bar{P}_m^{\bar{j}}\right]$.

(*case of* $N^J \xrightarrow{\lambda} N^{J \setminus \{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}} \setminus \{n\}} P_i^{\bar{j}} \mid \bar{P}_n^{\bar{j}}\right]$, $\lambda = \alpha$)

We know that $\left[P_n^{\bar{j}}\right] \xrightarrow{\lambda} \left[\bar{P}_n^{\bar{j}}\right]$, which since $\left[P_n^{\bar{j}}\right] \approx \left[Q_n^{\bar{j}}\right]$ gives us that there exists M' such that $\left[Q_n^{\bar{j}}\right] \stackrel{\lambda}{\Longrightarrow} M'$ and $\left[\bar{P}_n^{\bar{j}}\right] \approx M'$ which, considering Lemma 3.13, leads to there exists $\bar{Q}_n^{\bar{j}}$ such that $M' \equiv \left[\bar{Q}_n^{\bar{j}}\right]$.

Hence we can derive $M^J \stackrel{\lambda}{\Longrightarrow} M^{J \setminus \{\bar{j}\}} \mid \left[\prod_{i \in I_{\bar{j}} \setminus \{n\}} Q_i^{\bar{j}} \mid \bar{Q}_n^{\bar{j}}\right]$, which along with

$(N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n\}} P_i^{\bar{j}} \mid \bar{P}_n^{\bar{j}}\right], M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n\}} Q_i^{\bar{j}} \mid \bar{Q}_n^{\bar{j}}\right]) \in B$,

since $\left[\bar{P}_n^{\bar{j}}\right] \approx \left[\bar{Q}_n^{\bar{j}}\right]$, completes the proof for this case.

(*case of* $N^J \xrightarrow{\tau} N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n,m\}} P_i^{\bar{j}} \mid \bar{P}_n^{\bar{j}} \mid \bar{P}_m^{\bar{j}}\right]$)

Since a synchronization can take place we know that there exists a such that either $P_n^{\bar{j}} \equiv_p \bar{a}.R_1 \mid R_2$ and $P_m^{\bar{j}} \equiv_p a.R_3 \mid R_4$ or with the action and coaction placed the other way around, being the proofs analogous. Considering $P_n^{\bar{j}} \equiv_p \bar{a}.R_1 \mid R_2$ and $P_m^{\bar{j}} \equiv_p a.R_3 \mid R_4$ we have that $\left[P_n^{\bar{j}}\right] \xrightarrow{\bar{a}} [R_1 \mid R_2]$ and $\left[P_m^{\bar{j}}\right] \xrightarrow{a} [R_3 \mid R_4]$ being that $[R_1 \mid R_2] \equiv \left[\bar{P}_n^{\bar{j}}\right]$ and $[R_3 \mid R_4] \equiv \left[\bar{P}_m^{\bar{j}}\right]$. Since $\left[P_n^{\bar{j}}\right] \approx \left[Q_n^{\bar{j}}\right]$ we obtain that $\left[Q_n^{\bar{j}}\right] \xRightarrow{\bar{a}} M'$ and $\left[\bar{P}_n^{\bar{j}}\right] \approx M'$ which, considering Lemma 3.13, leads to there exists $\bar{Q}_n^{\bar{j}}$ such that $M' \equiv \left[\bar{Q}_n^{\bar{j}}\right]$. Also since $\left[P_m^{\bar{j}}\right] \approx \left[Q_m^{\bar{j}}\right]$ we get that $\left[Q_m^{\bar{j}}\right] \xRightarrow{a} M''$ and $\left[\bar{P}_m^{\bar{j}}\right] \approx M''$ from which, considering Lemma 3.13, we obtain that there exists $\bar{Q}_m^{\bar{j}}$ such that $M'' \equiv \left[\bar{Q}_n^{\bar{j}}\right]$. Hence we can derive that $M^J \xRightarrow{\tau} M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n,m\}} Q_i^{\bar{j}} \mid \bar{Q}_n^{\bar{j}} \mid \bar{Q}_m^{\bar{j}}\right]$. which along with

$(N^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n,m\}} P_i^{\bar{j}} \mid \bar{P}_n^{\bar{j}} \mid \bar{P}_m^{\bar{j}}\right], M^{J\setminus\{\bar{j}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{n,m\}} Q_i^{\bar{j}} \mid \bar{Q}_n^{\bar{j}} \mid \bar{Q}_m^{\bar{j}}\right]) \in B$

since $\left[\bar{P}_n^{\bar{j}}\right] \approx \left[\bar{Q}_n^{\bar{j}}\right]$ and $\left[\bar{P}_m^{\bar{j}}\right] \approx \left[\bar{Q}_m^{\bar{j}}\right]$, completes the proof for this case.

(*Transition triggered by a migration*)

We now consider that a migration takes place, for which we know that there are at least two sites involved, the origin and destination of the migrating process, hence we have that $\#J \geq 2$. We also know that there exist $\bar{j}, \bar{i}$ such that $P_{\bar{i}}^{\bar{j}} \equiv_p \mathbf{go}.R_2 \mid R_1$ from which we can obtain, considering $a \notin fn(\left[P_{\bar{i}}^{\bar{j}}\right] \mid \left[Q_{\bar{i}}^{\bar{j}}\right])$, that $\left[P_{\bar{i}}^{\bar{j}}\right] \xrightarrow{[a]} \left[P_{\bar{i}}^{\bar{j}}\right] \mid [a.\mathbf{nil}] \xrightarrow{\tau} [R_1] \mid [a.\mathbf{nil} \mid R_2]$. Since $\left[P_{\bar{i}}^{\bar{j}}\right] \approx \left[Q_{\bar{i}}^{\bar{j}}\right]$ we get that there exist $\bar{M}, M'$ such that $\left[Q_{\bar{i}}^{\bar{j}}\right] \xRightarrow{[a]} \bar{M} \xRightarrow{\tau} M'$ and $[R_1] \mid [a.\mathbf{nil} \mid R_2] \approx M'$, from which we can derive that there exist M_1', M_2' such that $M' \Rightarrow M_1' \mid M_2'$ and $[R_1] \approx M_1'$ and $[a.\mathbf{nil} \mid R_2] \approx M_2'$ which, considering Lemma 3.13, leads to there exist R_3, R_4 such that $M_1' \equiv [R_3]$ and $M_2' \equiv [R_4]$. From $[a.\mathbf{nil} \mid R_2] \approx [R_4]$ and $[a.\mathbf{nil} \mid R_2] \xrightarrow{a} [R_2]$ we conclude that there exists R_4' such that $[R_4] \xRightarrow{a} [R_4']$ and $[R_2] \approx [R_4']$. This allows us to conclude, since $a \notin fn(\left[Q_{\bar{i}}^{\bar{j}}\right])$, that $[R_4] \Rightarrow [a.\mathbf{nil} \mid R_4']$.

So we have that there exists $\bar{l} \in J$ such that

$N^J \xrightarrow{\tau} N^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{\bar{i}\}} P_i^{\bar{j}} \mid R_1\right] \mid \left[\prod_{i\in I_{\bar{l}}} P_i^{\bar{l}} \mid R_2\right]$.

Also from $\left[Q_{\bar{i}}^{\bar{j}}\right] \xRightarrow{[a]} [R_3] \mid [a.\mathbf{nil} \mid R_4']$ we can derive that

$M^J \xRightarrow{\tau} M^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{\bar{i}\}} Q_i^{\bar{j}} \mid R_3\right] \mid \left[\prod_{i\in I_{\bar{l}}} Q_i^{\bar{l}} \mid R_4'\right]$,

which along with

$$\begin{aligned}(\quad & N^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{\bar{i}\}} P_i^{\bar{j}} \mid R_1\right] \mid \left[\prod_{i\in I_{\bar{l}}} P_i^{\bar{l}} \mid R_2\right], \\ & M^{J\setminus\{\bar{j},\bar{l}\}} \mid \left[\prod_{i\in I_{\bar{j}}\setminus\{\bar{i}\}} Q_i^{\bar{j}} \mid R_3\right] \mid \left[\prod_{i\in I_{\bar{l}}} Q_i^{\bar{l}} \mid R_4'\right]) \in B\end{aligned}$$

since $[R_1] \approx [R_3]$ and $[R_2] \approx [R_4']$, completes the proof for this case.

(*Transition triggered by a grow transition*)

We have that $N^J \xrightarrow{[a]} N^J \mid [a.\mathbf{nil}]$ and we can directly derive that $M^J \xRightarrow{[a]} M^J \mid [a.\mathbf{nil}]$ which along with $(N^J \mid [a.\mathbf{nil}], M^J \mid [a.\mathbf{nil}]) \in B$, since $[a.\mathbf{nil}] \approx [a.\mathbf{nil}]$, completes the proof for this case.

(*Transition triggered by a failure*)

We have that there exists $\bar{J} \subseteq J$ such that $N^J \xrightarrow{\tau} N^{J\setminus\bar{J}}$. We can directly derive that $M^J \xrightarrow{\tau} M^{J\setminus\bar{J}}$ which along with $(N^{J\setminus\bar{J}}, M^{J\setminus\bar{J}}) \in B$ completes the proof for this case and also for this clause. ■

Proof of Lemma 3.15

We prove weak bisimilarity is a congruence.
Proof. We know that there exist J and $\{P^j \mid j \in J\}$ such that $N \equiv \prod_{j\in J}\left[P^j\right]$ which considering that $N \approx M$ and Lemma 3.13 gives us that there exist M' and $\{Q^j \mid j \in J\}$ such that $M \Rightarrow M' \equiv \prod_{j\in J}\left[Q^j\right]$ and for all $j \in J$ it is the case that $\left[P^j\right] \approx \left[Q^j\right]$. Also we know that there exists C such that $C\,[N] \equiv N \mid C$ and $C\,[M] \equiv M \mid C$ which, along with the fact that there exist I and $\{R^i \mid i \in I\}$ such that $C \equiv \prod_{i\in I}\left[R^i\right]$, gives us that $C\,[N] \equiv \prod_{j\in J}\left[P^j\right] \mid \prod_{i\in I}\left[R^i\right]$ and $C\,[M'] \equiv \prod_{j\in J}\left[Q^j\right] \mid \prod_{i\in I}\left[R^i\right]$, which along with for all $j \in J$ it is the case that $\left[P^j\right] \approx \left[Q^j\right]$ and for all $i \in I$ it is the case that $\left[R^i\right] \approx \left[R^i\right]$ and considering Lemma 3.14 gives us that $C\,[N] \approx C\,[M']$.

Let us now prove that $B \triangleq \{(C\,[N], C\,[M]) \mid N \approx M\}$ is a weak bisimulation by coinduction on the definition of weak bisimulation. Consider that there exist N_1, N_2 such that $C\,[N] \equiv N_1 \mid N_2$. We know that there exists M' such that $M \Rightarrow M'$ and $C\,[N] \approx C\,[M']$. From $C\,[N] \approx C\,[M']$ we have that there exist M_1', M_2' such that $C\,[M'] \Rightarrow M_1' \mid M_2'$ and $N_1 \approx M_1'$ and $N_2 \approx M_2'$. From $M \Rightarrow M'$ we can derive that $C\,[M] \Rightarrow C\,[M']$ and hence $C\,[M] \Rightarrow M_1' \mid M_2'$ which completes the proof for the first clause. Now let us consider that $C\,[N] \equiv \mathbf{0}$. We know that there exists C such that $C\,[N] \equiv N \mid C$ and since $C\,[N] \equiv \mathbf{0}$ we have that $C \equiv \mathbf{0}$ and $N \equiv \mathbf{0}$. From $N \equiv \mathbf{0}$ and $N \approx M$ we have that $M \equiv \mathbf{0}$ and since $C \equiv \mathbf{0}$ we get $C\,[M] \equiv \mathbf{0}$ thus proving the second clause. Now consider that there exist λ and $\bar{N}$ such that $C\,[N] \xrightarrow{\lambda} \bar{N}$. Again we know that there exists M' such that $M \Rightarrow M'$ and $C\,[N] \approx C\,[M']$. From $C\,[N] \approx C\,[M']$ and $C\,[N] \xrightarrow{\lambda} \bar{N}$ we have that there exists $\bar{M}$ such that $C\,[M'] \xRightarrow{\lambda} \bar{M}$ and $\bar{N} \approx \bar{M}$. From $M \Rightarrow M'$ we have that $C\,[M] \Rightarrow C\,[M']$ and since $C\,[M'] \xRightarrow{\lambda} \bar{M}$ we obtain $C\,[M] \xRightarrow{\lambda} \bar{M}$ which completes the proof for the third clause. ■

B.3 Proofs of full abstraction ($\approx = \cong$)

Proof of Lemma 3.16

We prove $\approx \subseteq \cong$.
Proof. We proceed by coinduction on the definition of weak reduction barbed congruence. Let us consider N, M such that $N \approx M$.

Consider now that there exists a such that $N{\downarrow_a}$. This means that there exist P_1, P_2, N' such that $N \equiv [a.P_1 \mid P_2] \mid N'$ from which we can derive $N \xrightarrow{a} [P_1 \mid P_2] \mid N'$ and hence, since $N \approx M$, we have that there exists $\bar{M}$ such that $M \xRightarrow{a} \bar{M}$ and $[P_1 \mid P_2] \mid N' \approx \bar{M}$. This gives us that there exist M_1, M_2 such that $M \Rightarrow M_1 \xrightarrow{a} M_2 \Rightarrow \bar{M}$ from which we have that there exist Q_1, Q_2, M' such that $M_1 \equiv [a.Q_1 \mid Q_2] \mid M'$ hence $M_1{\downarrow_a}$ and finally $M{\Downarrow_a}$ which completes the proof of the first clause.

Let us now consider that there exists N' such that $N \rightarrow N'$. We can derive that $N \xrightarrow{\tau} N'$ which, since $N \approx M$, gives us that there exists M' such that $M \xRightarrow{\tau} M'$ and $N' \approx M'$. From $M \xRightarrow{\tau} M'$ we get that $M \Rightarrow M'$ which along with $N' \approx M'$ completes the proof of the second clause.

Lemma 3.15 directly provides with the third clause. ■

Proof of Lemma 3.17

We prove $\approxeq \subseteq \approx$.

Proof. We proceed by coinduction on the definition of weak bisimulation. Let us consider N, M such that $N \approxeq M$.

Consider now that there exist N', N'' such that $N \equiv N' \mid N''$. We know that there exist J and $\{P^j \mid j \in J\}$ such that $N \equiv \prod_{j \in J}[P^j]$ and also that there exists $\bar{J} \subseteq J$ such that $N' \equiv \prod_{j \in \bar{J}}[P^j]$ and $N'' \equiv \prod_{j \in J \setminus \bar{J}}[P^j]$. From $N \equiv \prod_{j \in J}[P^j]$ and $N \approxeq M$, considering Lemma 3.6, we have that there exists $\{Q^j \mid j \in J\}$ such that $M \Rightarrow \prod_{j \in J}[Q^j]$ and for all $j \in J$ it is the case that $[P^j] \approxeq [Q^j]$. We can now write that $M \Rightarrow \prod_{j \in \bar{J}}[Q^j] \mid \prod_{j \in J \setminus \bar{J}}[Q^j]$. From the fact that for all $j \in \bar{J}$ it is the case that $[P^j] \approxeq [Q^j]$ and that for all $j \in J \setminus \bar{J}$ it is the case that $[P^j] \approxeq [Q^j]$, considering Lemma 3.5, we obtain $\prod_{j \in \bar{J}}[P^j] \approxeq \prod_{j \in \bar{J}}[Q^j]$ and $\prod_{j \in J \setminus \bar{J}}[P^j] \approxeq \prod_{j \in J \setminus \bar{J}}[Q^j]$, which completes the proof for this case.

Now consider that $N \equiv \mathbf{0}$. Lemma 3.6 provides directly that $M \equiv \mathbf{0}$.

Consider now that there exist λ and N' such that $N \xrightarrow{\lambda} N'$. We have that λ is either τ, or else there exists a such that $\lambda = \bar{a}$ or $\lambda = a$ or finally that there exists a such that $\lambda = [a]$. If $\lambda = \tau$ we have that $N \rightarrow N'$ and since $N \approxeq M$ we get that there exists M' such that $M \Rightarrow M'$, hence $M \xRightarrow{\tau} M'$, and $N' \approxeq M'$ which completes the proof for this case.

If there exists a such that $\lambda = \bar{a}$ we have that there exist P_1, P_2, N'' such that $N \equiv [\bar{a}.P_1 \mid P_2] \mid N''$ and $N' \equiv [P_1 \mid P_2] \mid N''$. Let us consider context

$C[\bullet] \triangleq [t.\mathbf{nil} \mid \mathbf{go}.a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil})] \mid \bullet$

with $t, f \notin \mathit{fn}(N \mid M)$ and $t \neq f$. We can easily derive that

$C[N] \rightarrow [a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil}) \mid \bar{a}.P_1 \mid P_2] \mid [t.\mathbf{nil}] \mid N'' \rightarrow$
$[a.(f.\mathbf{nil} \mid \bar{f}.\mathbf{nil}) \mid \bar{a}.P_1 \mid P_2] \mid N'' \rightarrow [f.\mathbf{nil} \mid \bar{f}.\mathbf{nil} \mid P_1 \mid P_2] \mid N'' \rightarrow [P_1 \mid P_2] \mid N''$

which, since $N \approxeq M$, gives us that $C[M] \Rightarrow^4 M'$ and $[P_1 \mid P_2] \mid N'' \approxeq M'$. Since barb f can be observed at some point but M' does not exhibit it, and recalling that $f \notin \mathit{fn}(M)$ we can conclude that there exist Q_1, Q_2, M'' such that $M \Rightarrow [\bar{a}.Q_1 \mid Q_2] \mid M'' \xrightarrow{\bar{a}} [Q_1 \mid Q_2] \mid M'' \Rightarrow M'$ which gives us that $M \xRightarrow{\bar{a}} M'$ which, since $N' \approxeq M'$ completes the proof for this case, being the proof for $\lambda = a$ analogous.

If $\lambda = [a]$ we have that there exists N' such that $N \xrightarrow{[a]} N'$ and $N' \equiv N \mid [a.\mathbf{nil}]$. We also know that $M \xrightarrow{[a]} M \mid [a.\mathbf{nil}]$ and hence $M \xRightarrow{[a]} M \mid [a.\mathbf{nil}]$ which given that $\approxeq$ is context closed directly gives us $N \mid [a.\mathbf{nil}] \approxeq M \mid [a.\mathbf{nil}]$ and completes the proof. ∎

B.4 Proofs of logical characterization of $\approx$

Proof of Lemma 3.22

We prove $\approx \subseteq =_{\mathcal{L}_w}$.

Proof. We prove that if $N \approx M$ then $\forall A \,.\, N \models A \implies M \models A$ (the symmetric is analogous to prove) and we do so by induction on the structure of formula A.

If $A = \mathbf{T}$ then $M \models A$.

If $A = \neg B$ we have that $N \not\models B$. Let us supose, aiming at a contradiction, that $M \models B$, which, by induction hypothesis, gives us $N \models B$ which is a contradiction and hence $M \not\models B$ and $M \models \neg B$.

If $A = B \wedge C$ then $N \models B$ and $N \models C$ from which, by induction hypothesis, we obtain $M \models B$ and $M \models C$ hence $M \models B \wedge C$.

If $A = B \Uparrow C$ then there exist N', N'' such that $N \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. From $N \approx M$ we get that there exists $\bar{M}$ such that $M \Rightarrow \bar{M}$ and $N' \mid N'' \approx \bar{M}$ which gives us that there exist M', M'' such that $M \Rightarrow M' \mid M''$ and $N' \approx M'$ and $N'' \approx M''$ from which, by induction hypothesis, we obtain $M' \models B$ and $M'' \models C$ and hence $M \models B \upuparrows C$.

If $A = \langle\!\langle \lambda \rangle\!\rangle B$ we have that there exists N' such that $N \overset{\lambda}{\Longrightarrow} N'$ and $N' \models B$. From $N \approx M$ we get that there exists M' such that $M \overset{\lambda}{\Longrightarrow} M'$ and $N' \approx M'$ which, by induction hypothesis, gives us that $M' \models B$ hence $M \models \langle \lambda \rangle B$. ∎

Proof of Lemma 3.23

We have $=_{\mathcal{L}_w} \subseteq \approx$.
Proof. We prove that $R \triangleq \{(N, M) \mid N =_{\mathcal{L}_w} M\}$ is a weak bisimulation by coinduction on the definition of weak bisimulation.

Let us consider that there exist N', M' such that $N \equiv N' \mid N''$. Let us consider $I \triangleq \{1, 2, \ldots, k\}, \{M'_i, M''_i \mid i \in I\}$ such that for all M', M'' such that if $M \Rightarrow M' \mid M''$ then there exists $i \in I$ such that $M' \equiv M'_i$ and $M'' \equiv M''_i$. Aiming at a contradiction, let us now assume that for all $i \in I$ it is the case that $N' \neq_{\mathcal{L}_w} M'_i$ or $N'' \neq_{\mathcal{L}_w} M''_i$ from which we can derive there exists $\{A_i, B_i \mid i \in I\}$ such that for all $i \in I$ it is the case that either $N' \models A_i$ and $M'_i \not\models A_i$ or $N'' \models B_i$ and $M''_i \not\models B_i$. We can now write that $N \models (\bigwedge_{i \in I} A_i) \upuparrows (\bigwedge_{i \in I} B_i)$ and since $N =_{\mathcal{L}_w} M$ we have that $M \models (\bigwedge_{i \in I} A_i) \upuparrows (\bigwedge_{i \in I} B_i)$ which gives us that there exist M', M'' such that $M \Rightarrow M' \mid M''$ and $M' \models (\bigwedge_{i \in I} A_i)$ and $M'' \models (\bigwedge_{i \in I} B_i)$. We also know that there exists $j \in I$ such that $M' \equiv M'_j$ and $M'' \equiv M''_j$ from which follows that $M'_j \models (\bigwedge_{i \in I} A_i)$ and $M''_j \models (\bigwedge_{i \in I} B_i)$ which provides with the intended contradiction since $M'_j \not\models A_j$ or $M''_j \not\models B_j$. We can therefore conclude that there exists $i \in I$ such that $N' =_{\mathcal{L}_w} M'_i$ and $N'' =_{\mathcal{L}_w} M''_i$, which completes the proof.

Now consider that $N \equiv \mathbf{0}$, hence $N \models \mathbf{0}$ from which we obtain $M \models \mathbf{0}$ ($N =_{\mathcal{L}_w} M$) thus $M \equiv \mathbf{0}$.

Let us now consider that there exist N' and λ such that $N \xrightarrow{\lambda} N'$, hence $N \models \langle\!\langle \lambda \rangle\!\rangle \mathbf{T}$ which since $N =_{\mathcal{L}_w} M$ gives us that $M \models \langle\!\langle \lambda \rangle\!\rangle \mathbf{T}$ and thus there exists M' such that $M \overset{\lambda}{\Longrightarrow} M'$. Let us consider $I \triangleq \{1, 2, \ldots, k\}, \{M'_i \mid i \in I\}$ such that for all M' such that if $M \overset{\lambda}{\Longrightarrow} M'$ then there exists $i \in I$ such that $M' \equiv M'_i$. Aiming at a contradiction, let us now assume that for all $i \in I$ it is the case that $N' \neq_{\mathcal{L}_w} M'_i$ which gives us that there exists $\{A_i \mid i \in I\}$ such that for all $i \in I$ it is the case that $N' \models A_i$ and $M'_i \not\models A_i$. We can now write that $N \models \langle\!\langle \lambda \rangle\!\rangle (\bigwedge_{i \in I} A_i)$ and since $N =_{\mathcal{L}_w} M$ we have $M \models \langle\!\langle \lambda \rangle\!\rangle (\bigwedge_{i \in I} A_i)$ from which we obtain that there exists M' such that $M \overset{\lambda}{\Longrightarrow} M'$ and $M' \models \bigwedge_{i \in I} A_i$. We also know that there exists $j \in I$ such that $M' \equiv M'_j$ which gives us $M'_j \models \bigwedge_{i \in I} A_i$ which contradicts $M'_j \not\models A_j$, hence $\exists_{i \in I} N' =_{\mathcal{L}_w} M'_i$ which completes the proof. ∎

C Auxiliar Lemmas to the proof of minimality (Theorem 3.26)

Lemma C.1 *For all A such that A is a minimal spatial logic formula not containing $\neg B$ we have that for all N such that $N \not\equiv \mathbf{0}$ and $N \models A$ then $N \mid [\mathbf{nil}] \models A$.*

Proof. We proceed by induction on the structure of formula A.

- (Case $B \wedge C$) We have that $N \models B$ and $N \models C$ hence, by induction hypothesis, $N \mid [\mathbf{nil}] \models B$ and $N \mid [\mathbf{nil}] \models C$ thus $N \mid [\mathbf{nil}] \models B \wedge C$.

- (Case $\mathbf{0}$) Since $N \not\equiv \mathbf{0}$ we have $N \not\models \mathbf{0}$.

- (Case $B \upuparrows C$) We have that there exist N', N'' such that $N \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. Since $N \mid [\mathbf{nil}] \rightarrow N$ we can derive that $N \mid [\mathbf{nil}] \Rightarrow N' \mid N''$ and hence $N \mid [\mathbf{nil}] \models B \upuparrows C$.
- (Case $\langle\langle\omega\rangle\rangle B$) We have that there exists N' such that $N \overset{\omega}{\Longrightarrow} N'$ and $N' \models B$. Since $N \mid [\mathbf{nil}] \rightarrow N$ we can derive that $N \mid [\mathbf{nil}] \overset{\omega}{\Longrightarrow} N'$ hence $N \mid [\mathbf{nil}] \models \langle\langle\omega\rangle\rangle B$.

■

Lemma C.2 *For all A such that A is a minimal spatial logic formula not containing $\neg B$ we have that for all N such that $N \mid [\mathbf{nil}] \models A$ then $N \models A$.*

Proof. Similar to the proof of Lemma C.1 ■

Lemma C.3 *For all A such that A is a minimal a spatial logic formula not containing $B \wedge C$ we have that for all N such that $N \mid [\mathbf{nil}] \models A$ then either $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models A$ or $N \models A$.*

Proof. We proceed by induction on the structure of formula A.

- (Case $\neg\neg B$) We have that $N \mid [\mathbf{nil}] \models B$ and hence by induction hypothesis we obtain that either $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models B$ or $N \models B$ which give us $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models \neg\neg B$ or $N \models \neg\neg B$.
- (Case $\neg\mathbf{0}$) We have that $N \mid [\mathbf{nil}] \models \neg\mathbf{0}$ and $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models \neg\mathbf{0}$.
- (Case $\neg(B \upuparrows C)$) We have that there exist no M', M'' such that $N \mid [\mathbf{nil}] \Rightarrow M' \mid M''$ and $M' \models B$ and $M'' \models C$. Let us assume, aiming at a contradiction, that $N \models B \upuparrows C$ hence there exist O', O'' such that $N \Rightarrow O' \mid O''$ and $O' \models B$ and $O'' \models C$. Since $N \mid [\mathbf{nil}] \rightarrow N$ we have that $N \mid [\mathbf{nil}] \Rightarrow O' \mid O''$ which gives our intended contradiction. We conclude $N \models \neg(B \upuparrows C)$.
- (Case $\neg(\langle\langle\omega\rangle\rangle B)$) We have that there exists no M' such that $N \mid [\mathbf{nil}] \overset{\omega}{\Longrightarrow} M'$ and $M' \models B$. Let us assume, aiming at a contradiction, that $N \models \langle\langle\omega\rangle\rangle B$ hence there exists O' such that $N \overset{\omega}{\Longrightarrow} O'$ and $O' \models B$. Since $N \mid [\mathbf{nil}] \rightarrow N$ we derive that $N \mid [\mathbf{nil}] \overset{\omega}{\Longrightarrow} O'$ which gives our intended contradiction. We conclude $N \models \neg(\langle\langle\omega\rangle\rangle B)$.
- (Case $\mathbf{0}$) We have that $N \mid [\mathbf{nil}] \not\models \mathbf{0}$.
- (Case $B \upuparrows C$) We have that there exist M', M'' such that $N \mid [\mathbf{nil}] \Rightarrow M' \mid M''$ and $M' \models B$ and $M'' \models C$. Since $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \rightarrow N \mid [\mathbf{nil}]$ we derive that $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \Rightarrow M' \mid M''$ thus $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models B \upuparrows C$.
- (Case $\langle\langle\omega\rangle\rangle B$) We have that there exists M' such that $N \mid [\mathbf{nil}] \overset{\omega}{\Longrightarrow} M'$ and $M' \models B$. Since $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \rightarrow N \mid [\mathbf{nil}]$ we derive that $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \overset{\omega}{\Longrightarrow} M'$ thus $N \mid [\mathbf{nil}] \mid [\mathbf{nil}] \models \langle\langle\omega\rangle\rangle B$.

■

Lemma C.4 *For all A such that A is a minimal spatial logic formula not containing* $\mathbf{0}$ *we have that if* $[\mathbf{nil}] \models A$ *then* $\mathbf{0} \models A$.

Proof. We abbreviate $\prod_{j \in J}[a_j.\mathbf{nil}] \mid \prod_{l \in L}[\mathbf{nil}]$ with $Z^{(J,L)}$ to simplify presentation. We consider $\mathbf{T}$ as our primitive formula and we proceed by showing that for all index sets J, L if $[\mathbf{nil}] \mid Z^{(J,L)} \models A$ then $Z^{(J,L)} \models A$ and we do so by induction on the structure of formula A.

- (Case $\mathbf{T}$) We have $[\mathbf{nil}] \mid Z^{(J,L)} \models \mathbf{T}$ and $Z^{(J,L)} \models \mathbf{T}$.

- (Case $\neg\mathbf{T}$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \not\models \neg\mathbf{T}$.

- (Case $\neg\neg B$) We have $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ and hence by induction hypothesis $Z^{(J,L)} \models B$ which gives us $Z^{(J,L)} \models \neg\neg B$.

- (Case $\neg(B \wedge C)$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \wedge C)$ and hence $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ or $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$. The former implies, by induction hypothesis, that $Z^{(J,L)} \models \neg B$ while the latter implies, by induction hypothesis, that $Z^{(J,L)} \models \neg C$ hence we can conclude that either $Z^{(J,L)} \models \neg B$ or $Z^{(J,L)} \models \neg C$ and thus $Z^{(J,L)} \models \neg(B \wedge C)$.

- (Case $\neg(B \Uparrow C)$) We have that there exist no N', N'' such that $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. Let us assume, aiming at a contradiction that there exist M', M'' such that $Z^{(J,L)} \Rightarrow M' \mid M''$ and $M' \models B$ and $M'' \models C$. Since $[\mathbf{nil}] \mid Z^{(J,L)} \rightarrow Z^{(J,L)}$ we can derive that $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow M' \mid M''$ which provides with our intended contradiction. Hence there exist no M', M'' such that $Z^{(J,L)} \Rightarrow M' \mid M''$ and $M' \models B$ and $M'' \models C$ so $Z^{(J,L)} \models \neg(B \Uparrow C)$.

- (Case $\neg(\langle\langle\omega\rangle\rangle B)$) We have that there exists no N' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} N'$ and $N' \models B$. Let us now assume, aiming at a contradiction, that there exists M' such that $Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ and $M' \models B$. Since $[\mathbf{nil}] \mid Z^{(J,L)} \rightarrow Z^{(J,L)}$ we can derive that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ which leads to our intended contradiction. So we conclude that there exists no M' such that $Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ and $M' \models B$ hence $Z^{(J,L)} \models \neg(\langle\langle\omega\rangle\rangle B)$.

- (*Case* $B \wedge C$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \models B \wedge C$ and hence $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ and $[\mathbf{nil}] \mid Z^{(J,L)} \models C$ which by induction hypothesis gives us that $Z^{(J,L)} \models B$ and $Z^{(J,L)} \models C$ and hence $Z^{(J,L)} \models B \wedge C$.

- (*Case* $B \Uparrow C$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \models B \Uparrow C$ which gives us that there exist N', N'' such that $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. We know that there exist $J', J'' \subseteq J$ and $L', L'' \subset L$ such that $J' \cap J'' = \emptyset$ and $L' \cap L'' = \emptyset$ and either $N' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$ or $N' \equiv Z^{(J',L')}$ and $N'' \equiv [\mathbf{nil}] \mid Z^{(J'',L'')}$ or finally $N' \equiv Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$. We derive that $Z^{(J,L)} \Rightarrow Z^{(J',L')} \mid Z^{(J'',L'')}$. For the first case we obtain by induction hypothesis that $Z^{(J',L')} \models B$ and we have that $Z^{(J'',L'')} \models C$ whilst for the second case we have that $Z^{(J',L')} \models B$ and by induction hypothesis we obtain that $Z^{(J'',L'')} \models C$ or finally for the third case we have that $Z^{(J',L')} \models B$ and $Z^{(J'',L'')} \models C$, so we conclude $Z^{(J,L)} \models B \Uparrow C$.

- (*Case* $\langle\langle b\rangle\rangle B$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle b\rangle\rangle B$ hence there exists N' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} N'$ and $N' \models B$. We know that there exists $i \in J$ such that $b = a_i$ so we have that there exist $J' \subseteq J \backslash \{i\}$ and $L' \subseteq L \cup \{k\}$, for $k \notin L$, such that either $N' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$ and we also know that $Z^{(J,L)} \stackrel{b}{\Longrightarrow} Z^{(J',L')}$. For the first case by induction hypothesis we obtain that $Z^{(J',L')} \models B$ and for the second case we have that $Z^{(J',L')} \models B$, so we conclude $Z^{(J,L)} \models \langle\langle b\rangle\rangle B$.

- (*Case* $\langle\langle \bar{b}\rangle\rangle B$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \not\models \langle\langle \bar{b}\rangle\rangle B$.

- (*Case* $\langle\langle [b]\rangle\rangle B$) We have that $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle [b]\rangle\rangle B$ hence there exists N' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} N'$ and $N' \models B$. We know that considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) we have that there exist $J' \subseteq J \cup J''$ and $L' \subseteq L$ such that either $N' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$ and we also know that $Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} Z^{(J',L')}$. For the first case by induction hypothesis we obtain that $Z^{(J',L')} \models B$ whilst for the second case we have that $Z^{(J',L')} \models B$, so we have that $Z^{(J,L)} \models \langle\langle [b] \rangle\rangle B$.

■

Lemma C.5 *For all A such that A is a minimal spatial logic formula not containing* $\mathbf{0}$ *we have that if* $\mathbf{0} \models A$ *then* $[\mathbf{nil}] \models A$*.*

Proof. Similar to the proof of Lemma C.4. ■

Lemma C.6 *For all A such that A is a minimal spatial logic formula not containing $B \upharpoonleft\upharpoonright C$ we have that if* $[\mathbf{nil}] \mid [\mathbf{nil}] \models A$ *then* $[\mathbf{nil}] \models A$*.*

Proof. We abbreviate $\prod_{j \in J}[a_j.\mathbf{nil}] \mid \prod_{l \in L}[\mathbf{nil}]$ with $Z^{(J,L)}$ to simplify presentation. We proceed by showing that for all index sets J, L if $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models A$ then $[\mathbf{nil}] \mid Z^{(J,L)} \models A$ and we do so by induction on the structure of formula A.

- (Case $\neg\neg B$) We have $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models B$ and hence by induction hypothesis $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ which gives us $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg\neg B$.
- (Case $\neg(B \wedge C)$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \wedge C)$ and hence $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ or $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$. The former implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ while the latter implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$ hence we can conclude that either $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ or $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$ thus $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \wedge C)$.
- (Case $\neg\mathbf{0}$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \neg\mathbf{0}$ and $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg\mathbf{0}$.
- (Case $\neg(\langle\langle \omega \rangle\rangle B)$) We have that there exists no N' such that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} N'$ and $N' \models B$. Let us now assume, aiming at a contradiction, that there exists M' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ and $M' \models B$. Since $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \rightarrow [\mathbf{nil}] \mid Z^{(J,L)}$ we can derive that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ which leads to our intended contradiction. So we conclude there exists no M' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} M'$ and $M' \models B$ hence $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(\langle\langle \omega \rangle\rangle B)$.
- (*Case* $B \wedge C$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models B \wedge C$ and hence $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models B$ and $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models C$ which by induction hypothesis gives us that $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ and $[\mathbf{nil}] \mid Z^{(J,L)} \models C$ and hence $[\mathbf{nil}] \mid Z^{(J,L)} \models B \wedge C$.
- (*Case* $\mathbf{0}$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \not\models \mathbf{0}$.
- (*Case* $\langle\langle b \rangle\rangle B$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle b \rangle\rangle B$ hence there exists N' such that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} N'$ and $N' \models B$. We know that there exists $i \in J$ such that $b = a_i$ so we have that there exist $J' \subseteq J \backslash \{i\}$ and $L' \subseteq L \cup \{k\}$, for $k \notin L$, such that either $N' \equiv [\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$ and we also know that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} [\mathbf{nil}] \mid Z^{(J',L')}$ and $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} Z^{(J',L')}$. For the first case by induction hypothesis we obtain that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ and for the second case we have that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ and also for the third case we have that $Z^{(J',L')} \models B$, so we conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle b \rangle\rangle B$.
- (*Case* $\langle\langle \bar{b} \rangle\rangle B$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \not\models \langle\langle \bar{b} \rangle\rangle B$.

- (*Case* $\langle\langle[b]\rangle\rangle B$) We have that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle[b]\rangle\rangle B$ hence there exists N' such that $[\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} N'$ and $N' \models B$. We know that considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) we have that there exist $J' \subseteq J \cup J''$ and $L' \subseteq L$ such that either $N' \equiv [\mathbf{nil}] \mid [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$ and we also know that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} [\mathbf{nil}] \mid Z^{(J',L')}$ and $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} Z^{(J',L')}$. For the first case by induction hypothesis we obtain that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ whilst for the second case we have that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ and also for the third case we have that $Z^{(J',L')} \models B$, so we conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\langle[b]\rangle\rangle B$.

■

Lemma C.7 *For all A such that A is a minimal spatial logic formula not containing $B \upuparrows C$ we have that if* $[\mathbf{nil}] \models A$ *then* $[\mathbf{nil}] \mid [\mathbf{nil}] \models A$.

Proof. Similar to the proof of Lemma C.6 ■

Lemma C.8 *For all A such that A is a minimal spatial logic formula not containing $\langle\langle b\rangle\rangle B$ we have that if* $[a.\mathbf{nil}] \models A$ *then* $[\mathbf{nil}] \models A$.

Proof. We abbreviate $\prod_{j\in J}[a_j.\mathbf{nil}]$ with Z^J to simplify presentation. We proceed by showing that for any index set J if $[a.\mathbf{nil}] \mid Z^J \models A$ then $[\mathbf{nil}] \mid Z^J \models A$ and we do so by induction on the structure of formula A.

- (Case $\neg\neg B$) We have $[a.\mathbf{nil}] \mid Z^J \models B$ and hence by induction hypothesis $[\mathbf{nil}] \mid Z^J \models B$ which gives us $[\mathbf{nil}] \mid Z^J \models \neg\neg B$.

- (Case $\neg(B \wedge C)$) We have that $[a.\mathbf{nil}] \mid Z^J \models \neg(B \wedge C)$ and hence $[a.\mathbf{nil}] \mid Z^J \models \neg B$ or $[a.\mathbf{nil}] \mid Z^J \models \neg C$. The former implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^J \models \neg B$ while the latter implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^J \models \neg C$ hence we can conclude that either $[\mathbf{nil}] \mid Z^J \models \neg B$ or $[\mathbf{nil}] \mid Z^J \models \neg C$ thus $[\mathbf{nil}] \mid Z^J \models \neg(B \wedge C)$.

- (Case $\neg\mathbf{0}$) We have that $[a.\mathbf{nil}] \mid Z^J \models \neg\mathbf{0}$ and $[\mathbf{nil}] \mid Z^J \models \neg\mathbf{0}$.

- (Case $\neg(B \upuparrows C)$) We have that there exist no N', N'' such that $[a.\mathbf{nil}] \mid Z^J \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$, hence for all N', N'' such that $[a.\mathbf{nil}] \mid Z^J \Rightarrow N' \mid N''$ either $N' \models \neg B$ or $N'' \models \neg C$. We know that for all M', M'' such that $[\mathbf{nil}] \mid Z^J \Rightarrow M' \mid M''$ we have that either $M' \equiv [\mathbf{nil}] \mid Z^{J'}$ and $M'' \equiv Z^{J''}$ or $M' \equiv Z^{J'}$ and $M'' \equiv [\mathbf{nil}] \mid Z^{J''}$ or finally $M' \equiv Z^{J'}$ and $M'' \equiv Z^{J''}$. For the first case since we have that $[a.\mathbf{nil}] \mid Z^J \Rightarrow [a.\mathbf{nil}] \mid Z^{J'} \mid Z^{J''}$ we obtain that if $[a.\mathbf{nil}] \mid Z^{J'} \models \neg B$ then by induction hypothesis we have that $[\mathbf{nil}] \mid Z^{J'} \models \neg B$ and if $Z^{J''} \models \neg C$ then the result is immediate. For the second case since we have that $[a.\mathbf{nil}] \mid Z^J \Rightarrow Z^{J'} \mid [a.\mathbf{nil}] \mid Z^{J''}$ we obtain that if $Z^{J'} \models \neg B$ then the result is immediate and if $[a.\mathbf{nil}] \mid Z^{J''} \models \neg C$ then by induction hypothesis we have $[\mathbf{nil}] \mid Z^{J''} \models \neg C$ or finally for the third case since we have that $[a.\mathbf{nil}] \mid Z^J \Rightarrow Z^{J'} \mid Z^{J''}$ if either $Z^{J'} \models \neg B$ or $Z^{J''} \models \neg C$ the result is immediate. We conclude $[\mathbf{nil}] \mid Z^J \models \neg(B \upuparrows C)$.

- (Case $\neg(\langle\langle\omega\rangle\rangle B)$) We have that there exists no N' such that $[a.\mathbf{nil}] \mid Z^J \stackrel{\omega}{\Longrightarrow} N'$ and $N' \models B$, hence for all N' such that $[a.\mathbf{nil}] \mid Z^J \stackrel{\omega}{\Longrightarrow} N'$ it is the case that $N' \models \neg B$. If $\omega = \bar{b}$ then we have that $[\mathbf{nil}] \mid Z^J \models \neg(\langle\langle\bar{b}\rangle\rangle B)$ since there are no outputs in the network. If $\omega = [b]$ we know that for all M' such that $[\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} M'$ we have that considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) there exists $J' \subseteq J \cup J''$ such that either $M' \equiv [\mathbf{nil}] \mid Z^{J'}$ or $M' \equiv Z^{J'}$. For the first case since we know that $[a.\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} [a.\mathbf{nil}] \mid Z^{J'}$ and $[a.\mathbf{nil}] \mid Z^{J'} \models \neg B$ then by induction hypothesis we obtain that $[\mathbf{nil}] \mid Z^{J'} \models \neg B$ whilst for the second case since we know $[a.\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} Z^{J'}$ we have that $Z^{J'} \models \neg B$ and the result is immediate. So we conclude $[\mathbf{nil}] \mid Z^J \models \neg(\langle\langle [b] \rangle\rangle B)$.

- (*Case* $B \wedge C$) We have that $[a.\mathbf{nil}] \mid Z^J \models B \wedge C$ and hence $[a.\mathbf{nil}] \mid Z^J \models B$ and $[a.\mathbf{nil}] \mid Z^J \models C$ which by induction hypothesis gives us that $[\mathbf{nil}] \mid Z^J \models B$ and $[\mathbf{nil}] \mid Z^J \models C$ and hence $[\mathbf{nil}] \mid Z^J \models B \wedge C$.

- (*Case* $\mathbf{0}$) We have that $[a.\mathbf{nil}] \mid Z^J \not\models \mathbf{0}$.

- (*Case* $B \upharpoonleft\upharpoonright C$) We have that $[a.\mathbf{nil}] \mid Z^J \models B \upharpoonleft\upharpoonright C$ which gives us that there exist N', N'' such that $[a.\mathbf{nil}] \mid Z^J \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. We know that there exist $J', J'' \subseteq J$ such that $J' \cap J'' = \emptyset$ and either $N' \equiv [a.\mathbf{nil}] \mid Z^{J'}$ and $N'' \equiv Z^{J''}$ or $N' \equiv Z^{J'}$ and $N'' \equiv [a.\mathbf{nil}] \mid Z^{J''}$ or $N' \equiv Z^{J'}$ and $N'' \equiv Z^{J''}$. For the first case we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{J'} \models B$ and we have that $Z^{J''} \models C$ which along with $[\mathbf{nil}] \mid Z^J \Rightarrow [\mathbf{nil}] \mid Z^{J'} \mid Z^{J''}$ completes the proof for this case. For the second case we have that $Z^{J'} \models B$ and we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{J''} \models C$ which along with $[\mathbf{nil}] \mid Z^J \Rightarrow Z^{J'} \mid [\mathbf{nil}] \mid Z^{J''}$ completes the proof for this case. Finally for the third case since $[\mathbf{nil}] \mid Z^J \Rightarrow Z^{J'} \mid Z^{J''}$ the result is immediate. So we conclude $[\mathbf{nil}] \mid Z^J \models B \upharpoonleft\upharpoonright C$.

- (*Case* $\langle\langle \bar{b} \rangle\rangle B$) We have that $[a.\mathbf{nil}] \mid Z^J \not\models \langle\langle \bar{b} \rangle\rangle B$.

- (*Case* $\langle\langle [b] \rangle\rangle B$) We have that $[a.\mathbf{nil}] \mid Z^J \models \langle\langle [b] \rangle\rangle B$ hence there exists N' such that $[a.\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} N'$ and $N' \models B$. We know that considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) we have that there exists $J' \subseteq J \cup J''$ such that either $N' \equiv [a.\mathbf{nil}] \mid Z^{J'}$ or $N' \equiv Z^{J'}$. For the first case we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{J'} \models B$ which since $[\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} [\mathbf{nil}] \mid Z^{J'}$ completes the proof for this case. For the second case since $[\mathbf{nil}] \mid Z^J \stackrel{[b]}{\Longrightarrow} Z^{J'}$ the result is immediate. We conclude $[\mathbf{nil}] \mid Z^J \models \langle\langle [b] \rangle\rangle B$.

■

Lemma C.9 *For all A such that A is a minimal spatial logic formula not containing $\langle\langle b \rangle\rangle B$ we have that if $[\mathbf{nil}] \models A$ then $[a.\mathbf{nil}] \models A$.*

Proof. Analogous to the proof of Lemma C.8. ■

Lemma C.10 *For all A such that A is a minimal spatial logic formula not containing $\langle\langle \bar{b} \rangle\rangle B$ we have that if $[\bar{a}.\mathbf{nil}] \models A$ then $[\mathbf{nil}] \models A$.*

Proof. We abbreviate $\prod_{j \in J} [a_j.\mathbf{nil}] \mid \prod_{l \in L} [\mathbf{nil}]$ with $Z^{(J,L)}$ to simplify presentation. We proceed by showing that for all index sets J, L if $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models A$ then $[\mathbf{nil}] \mid Z^{(J,L)} \models A$ and we do so by induction on the structure of formula A.

- (Case $\neg\neg B$) We have $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models B$ and hence by induction hypothesis $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ which gives us $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg\neg B$.

- (Case $\neg(B \wedge C)$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \wedge C)$ and hence $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ or $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$. The former implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ while the latter implies, by induction hypothesis, that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$ hence we can conclude that either $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg B$ or $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg C$ thus $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \wedge C)$.

- (Case $\neg\mathbf{0}$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \neg\mathbf{0}$ and $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg\mathbf{0}$.

- (Case $\neg(B \upuparrows C)$) We have that there exist no N', N'' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$, hence for all N', N'' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow N' \mid N''$ either $N' \models \neg B$ or $N'' \models \neg C$. We can derive that for all M', M'' such that $[\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow M' \mid M''$ it is either the case that $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ and $M'' \equiv Z^{(J'',L'')}$ or $M' \equiv Z^{(J',L')}$ and $M'' \equiv [\mathbf{nil}] \mid Z^{(J'',L'')}$ or finally $M' \equiv Z^{(J',L')}$ and $M'' \equiv Z^{(J'',L'')}$. If $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ and $M'' \equiv Z^{(J'',L'')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')} \mid Z^{(J'',L'')}$ we have that either $[\bar{a}.\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$ or $Z^{(J'',L'')} \models \neg C$. The former gives us by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$ while the latter immediately provides with the result. If $M' \equiv Z^{(J',L')}$ and $M'' \equiv [\mathbf{nil}] \mid Z^{(J'',L'')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow Z^{(J',L')} \mid [\bar{a}.\mathbf{nil}] \mid Z^{(J'',L'')}$ we have that either $Z^{(J',L')} \models \neg B$ or $[\bar{a}.\mathbf{nil}] \mid Z^{(J'',L'')} \models \neg C$. The former immediately provides with the result while the latter gives us by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J'',L'')} \models \neg C$. If $M' \equiv Z^{(J',L')}$ and $M'' \equiv Z^{(J'',L'')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rrightarrow Z^{(J',L')} \mid Z^{(J'',L'')}$ we immediately have that either $Z^{(J',L')} \models \neg B$ or $Z^{(J'',L'')} \models \neg C$. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(B \upuparrows C)$.

- (Case $\neg(\langle\!\langle \omega \rangle\!\rangle B)$) We have that there exists no N' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} N'$ and $N' \models B$, hence for all N' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{\omega}{\Longrightarrow} N'$ it is the case that $N' \models \neg B$. If $\omega = b$ and for all $i \in J$ it is the case that $b \neq a_i$ then we immediately have that $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(\langle\!\langle b \rangle\!\rangle B)$ since the transition can not occur. If $\omega = b$ and there exists $i \in J$ such that $b = a_i$ then we know that for all M' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} M'$ we have that there exist $J' \subseteq J \backslash \{i\}$ and $L' \subseteq L \cup \{k\}$, for $k \notin L$, such that either $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $M' \equiv Z^{(J',L')}$. For $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ and $[\bar{a}.\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$ we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$. For $M' \equiv Z^{(J',L')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} Z^{(J',L')}$ we immediately have that $Z^{(J',L')} \models \neg B$. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(\langle\!\langle b \rangle\!\rangle B)$.

  If $\omega = [b]$ then we know that for all M' such that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} M'$ and considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) we have that there exist $J' \subseteq J \cup J''$ and $L' \subseteq L$ such that either $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ or $M' \equiv Z^{(J',L')}$. For $M' \equiv [\mathbf{nil}] \mid Z^{(J',L')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ and $[\bar{a}.\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$ we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J',L')} \models \neg B$. For $M' \equiv Z^{(J',L')}$ since we know that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} Z^{(J',L')}$ we immediately have that $Z^{(J',L')} \models \neg B$. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \neg(\langle\!\langle [b] \rangle\!\rangle B)$.

- (*Case* $B \wedge C$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models B \wedge C$ and hence $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models B$ and $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models C$ which by induction hypothesis gives us that $[\mathbf{nil}] \mid Z^{(J,L)} \models B$ and $[\mathbf{nil}] \mid Z^{(J,L)} \models C$ and hence $[\mathbf{nil}] \mid Z^{(J,L)} \models B \wedge C$.

- (*Case* $\mathbf{0}$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \not\models \mathbf{0}$.

- (*Case* $B \Uparrow C$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models B \Uparrow C$ which gives us that there exist N', N'' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. We know that there exist $J', J'' \subseteq J$ and $L', L'' \subset L$ such that $J' \cap J'' = \emptyset$ and $L' \cap L'' = \emptyset$ and either $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$ or $N' \equiv Z^{(J',L')}$ and $N'' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J'',L'')}$ or finally $N' \equiv Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$. If $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$ then by induction hypothesis we obtain $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ and we have $Z^{(J'',L'')} \models C$ which along with $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow [\mathbf{nil}] \mid Z^{(J',L')} \mid Z^{(J'',L'')}$ completes the proof for this case. If $N' \equiv Z^{(J',L')}$ and $N'' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J'',L'')}$ then we have $Z^{(J',L')} \models B$ and by induction hypothesis we obtain $[\mathbf{nil}] \mid Z^{(J'',L'')} \models C$ which along with $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow Z^{(J',L')} \mid [\mathbf{nil}] \mid Z^{(J'',L'')}$ completes the proof for this case. Finally if $N' \equiv Z^{(J',L')}$ and $N'' \equiv Z^{(J'',L'')}$ then we have $Z^{(J',L')} \models B$ and $Z^{(J'',L'')} \models C$ which along with $[\mathbf{nil}] \mid Z^{(J,L)} \Rightarrow Z^{(J',L')} \mid Z^{(J'',L'')}$ completes the proof for this case. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models B \Uparrow C$.

- (*Case* $\langle\!\langle b \rangle\!\rangle B$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \langle\!\langle b \rangle\!\rangle B$ hence there exists N' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} N'$ and $N' \models B$. We know that there exists $i \in J$ such that $b = a_i$ so we have that there exist $J' \subseteq J \backslash \{i\}$ and $L' \subseteq L \cup \{k\}$, for $k \notin L$, such that either $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$. If $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ which along with $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} [\mathbf{nil}] \mid Z^{(J',L')}$ completes the proof for this case. If $N' \equiv Z^{(J',L')}$ since we know that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{b}{\Longrightarrow} Z^{(J',L')}$ the result is immediate. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\!\langle b \rangle\!\rangle B$.

- (*Case* $\langle\!\langle [b] \rangle\!\rangle B$) We have that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \models \langle\!\langle [b] \rangle\!\rangle B$ hence there exists N' such that $[\bar{a}.\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} N'$ and $N' \models B$. We know that considering J'' such that $J \cap J'' = \emptyset$ and $\{a_j \mid j \in J''\} = \{b\}$ (note that we just separate the index set to simplify presentation, we do not enforce any conditions on b) we have that there exist $J' \subseteq J \cup J''$ and $L' \subseteq L$ such that either $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ or $N' \equiv Z^{(J',L')}$. If $N' \equiv [\bar{a}.\mathbf{nil}] \mid Z^{(J',L')}$ we obtain by induction hypothesis that $[\mathbf{nil}] \mid Z^{(J',L')} \models B$ which along with $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} [\mathbf{nil}] \mid Z^{(J',L')}$ completes the proof for this case. If $N' \equiv Z^{(J',L')}$ since we know that $[\mathbf{nil}] \mid Z^{(J,L)} \stackrel{[b]}{\Longrightarrow} Z^{(J',L')}$ the result is immediate. We conclude $[\mathbf{nil}] \mid Z^{(J,L)} \models \langle\!\langle [b] \rangle\!\rangle B$.

■

Lemma C.11 *For all A such that A is a minimal spatial logic formula not containing $\langle\!\langle \bar{b} \rangle\!\rangle B$ we have that if $[\mathbf{nil}] \models A$ then $[\bar{a}.\mathbf{nil}] \models A$.*

Proof. Analogous to the proof of Lemma C.10. ■

Lemma C.12 *For all A such that A is a minimal spatial logic formula not containing $\langle\!\langle [a] \rangle\!\rangle B$ we have that if $[\mathbf{go}.b.\mathbf{nil}] \models A$ then $[\mathbf{nil}] \models A$.*

Proof. We proceed by induction on the structure of the formula A.

- (*Case* $\neg\neg B$) We have that $[\mathbf{go}.b.\mathbf{nil}] \models B$ and hence by induction hypothesis we obtain $[\mathbf{nil}] \models B$ and thus $[\mathbf{nil}] \models \neg\neg B$.

- (*Case* $\neg(B \wedge C)$) We have that $[\mathbf{go}.b.\mathbf{nil}] \models \neg(B \wedge C)$ and hence $[\mathbf{go}.b.\mathbf{nil}] \models \neg B$ or $[\mathbf{go}.b.\mathbf{nil}] \models \neg C$. The former implies by induction hypothesis that $[\mathbf{nil}] \models \neg B$ and the latter implies by induction hypothesis that $[\mathbf{nil}] \models \neg C$ hence either $[\mathbf{nil}] \models \neg B$ or $[\mathbf{nil}] \models \neg C$ thus $[\mathbf{nil}] \models \neg(B \wedge C)$.

- (*Case* $\neg\mathbf{0}$) We have that $[\mathbf{go}.b.\mathbf{nil}] \models \neg\mathbf{0}$ and $[\mathbf{nil}] \models \neg\mathbf{0}$.

- (*Case* $\neg(B \upharpoonleft\upharpoonright C)$) We have that $[\mathbf{go}.b.\mathbf{nil}] \models \neg(B \upharpoonleft\upharpoonright C)$ and hence there exist no N', N'' such that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$, which amounts to say that for all N', N'' such that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow N' \mid N''$ it is the case that either $N' \models \neg B$ or $N'' \models \neg C$. We have that for all M', M'' such that $[\mathbf{nil}] \Rightarrow M' \mid M''$ either $M' \equiv [\mathbf{nil}]$ and $M'' \equiv \mathbf{0}$ or $M' \equiv \mathbf{0}$ and $M'' \equiv [\mathbf{nil}]$ or finally $M' \equiv \mathbf{0}$ and $M'' \equiv \mathbf{0}$. For $M' \equiv [\mathbf{nil}]$ and $M'' \equiv \mathbf{0}$ we know that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow [\mathbf{go}.b.\mathbf{nil}] \mid \mathbf{0}$ and either $[\mathbf{go}.b.\mathbf{nil}] \models \neg B$ or $\mathbf{0} \models \neg C$. If $[\mathbf{go}.b.\mathbf{nil}] \models \neg B$ then by induction hypothesis we have that $[\mathbf{nil}] \models \neg B$ and if $\mathbf{0} \models \neg C$ then the result is immediate. For $M' \equiv \mathbf{0}$ and $M'' \equiv [\mathbf{nil}]$ we know that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow \mathbf{0} \mid [\mathbf{go}.b.\mathbf{nil}]$ and either $\mathbf{0} \models \neg B$ or $[\mathbf{go}.b.\mathbf{nil}] \models \neg C$. If $\mathbf{0} \models \neg B$ then the result is immediate and if $[\mathbf{go}.b.\mathbf{nil}] \models \neg C$ then by induction hypothesis we have that $[\mathbf{nil}] \models \neg C$. Finally for $M' \equiv \mathbf{0}$ and $M'' \equiv \mathbf{0}$ we know that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow \mathbf{0} \mid \mathbf{0}$ and either $\mathbf{0} \models \neg B$ or $\mathbf{0} \models \neg C$ hence the result is immediate. We conclude $[\mathbf{nil}] \models \neg(B \upharpoonleft\upharpoonright C)$.

- (*Case* $\neg(\langle\langle\omega\rangle\rangle B)$) Since there are no transitions (recall that $[a]$ is not observable) we immediately have that $[\mathbf{go}.b.\mathbf{nil}] \models \neg(\langle\langle\omega\rangle\rangle B)$ and $[\mathbf{nil}] \models \neg(\langle\langle\omega\rangle\rangle B)$.

- (*Case* $B \wedge C$) We have that $[\mathbf{go}.b.\mathbf{nil}] \models B \wedge C$ and hence $[\mathbf{go}.b.\mathbf{nil}] \models B$ and $[\mathbf{go}.b.\mathbf{nil}] \models C$ which by induction hypothesis gives us that $[\mathbf{nil}] \models B$ and $[\mathbf{nil}] \models C$ and hence $[\mathbf{nil}] \models B \wedge C$.

- (*Case* $\mathbf{0}$) We have that $[\mathbf{go}.b.\mathbf{nil}] \not\models \mathbf{0}$.

- (*Case* $B \upharpoonleft\upharpoonright C$) We have that $\exists N', N''$ such that $[\mathbf{go}.b.\mathbf{nil}] \Rightarrow N' \mid N''$ and $N' \models B$ and $N'' \models C$. We know that either $N' \equiv [\mathbf{go}.b.\mathbf{nil}]$ and $N'' \equiv \mathbf{0}$ or $N' \equiv \mathbf{0}$ and $N'' \equiv [\mathbf{go}.b.\mathbf{nil}]$ or finally $N' \equiv \mathbf{0}$ and $N'' \equiv \mathbf{0}$. If $N' \equiv [\mathbf{go}.b.\mathbf{nil}]$ and $N'' \equiv \mathbf{0}$ then by induction hypothesis we obtain $[\mathbf{nil}] \models B$ and we have that $\mathbf{0} \models C$ which along with $[\mathbf{nil}] \Rightarrow [\mathbf{nil}] \mid \mathbf{0}$ completes the proof for this case. If $N' \equiv \mathbf{0}$ and $N'' \equiv [\mathbf{go}.b.\mathbf{nil}]$ then we have that $\mathbf{0} \models B$ and by induction hypothesis we obtain $[\mathbf{nil}] \models C$ which along with $[\mathbf{nil}] \Rightarrow \mathbf{0} \mid [\mathbf{nil}]$ completes the proof for this case. If $N' \equiv \mathbf{0}$ and $N'' \equiv \mathbf{0}$ then since $[\mathbf{nil}] \Rightarrow \mathbf{0} \mid \mathbf{0}$ the result is immediate. We conclude $[\mathbf{nil}] \models B \upharpoonleft\upharpoonright C$.

- (*Case* $\langle\langle\omega\rangle\rangle B$) We have that $[\mathbf{go}.b.\mathbf{nil}] \not\models \langle\langle\omega\rangle\rangle B$, since there are no possible transitions. Note that since $[a]$ is not observable the migration can not occur.

■

Lemma C.13 *For all A such that A is a minimal spatial logic formula not containing $\langle\langle [a] \rangle\rangle B$ we have that if $[\mathbf{nil}] \models A$ then $[\mathbf{go}.b.\mathbf{nil}] \models A$.*

Proof. Analogous to proof of Lemma C.12 ■